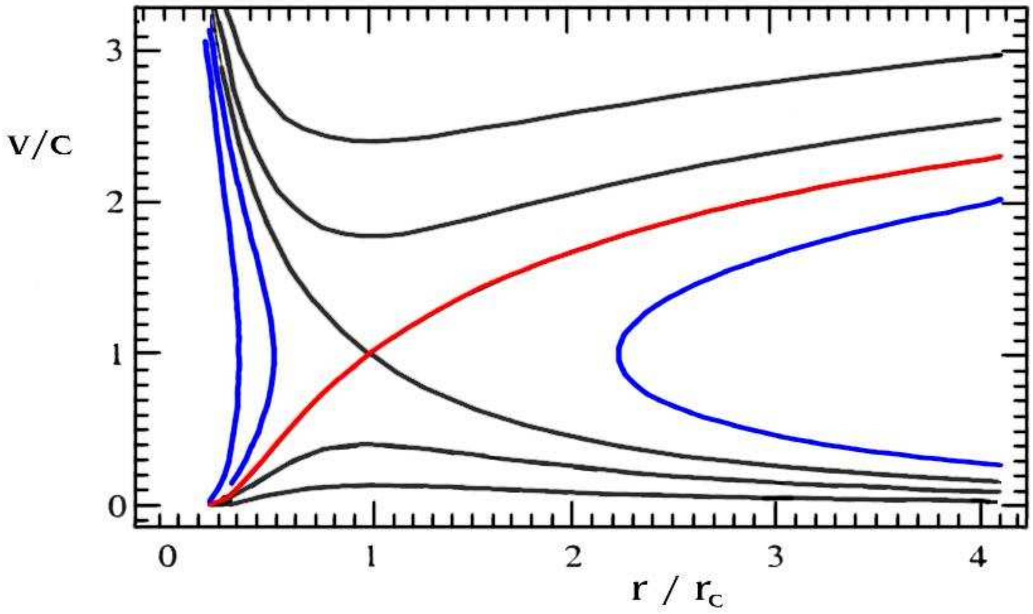
Hydrodynamic Parker model

spherical: $\mathbf{v}(\mathbf{r}) = v(r)\ \mathbf{e}_\mathrm{r}$
isothermal: $P = (k_\mathrm{B}/m)\, T_0\, \rho = c^2\, \rho$

$$\frac{d}{dr}(r^2 \rho\, v) = 0 \quad \wedge \quad v\,\frac{dv}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM_\mathrm{Sun}}{r^2}$$

$$\Rightarrow \boxed{\frac{v^2 - c^2}{v}\,\frac{dv}{dr} = \frac{2c^2}{r} - \frac{GM_\mathrm{Sun}}{r^2}}$$

critical (sonic) point at $r = r_c := \dfrac{GM_\mathrm{Sun}}{2c^2}$



discard as unphysical   disagrees w/observation

SOLAR WIND

$T = 10^6$ K $\Rightarrow c = 130$ km/s $\Rightarrow r_c = 5.8\ R_\mathrm{Sun}$

$$\left.\begin{array}{l} r \gg r_c \\ v \gg c \end{array}\right\} \Rightarrow \frac{v(r)}{2c} \approx \sqrt{\ln\left(\frac{r}{r_c}\right)},\quad \frac{\rho(r)}{c} \propto \frac{\Phi}{r^2}\left[\ln\left(\frac{r}{r_c}\right)\right]^{-\frac{1}{2}}$$

Suggestion for Basic Equations:

$$
\begin{aligned}
\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\rho\left[\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] &= -\nabla p - \rho G M_{\mathsf{Sun}}/r^2\ \mathbf{e}_r \\
&\quad -\nabla p_{\mathsf{W}} + (\mathbf{j}\times\mathbf{B})/c \\
&\quad +\nabla\cdot(\nu_{\mathsf{R}}\rho\nabla\cdot\mathbf{v})
\end{aligned}
$$

$$
\frac{n k_{\mathsf{B}}}{\gamma-1}\left[\partial_t T_{\mathsf{p}} + (\mathbf{v}\cdot\nabla)T_{\mathsf{p}}\right] + n k_{\mathsf{B}} T_{\mathsf{p}}(\nabla\cdot\mathbf{v}) =
$$

$$
\frac{n k_{\mathsf{B}}}{\gamma-1}\nu_{\mathsf{ep}}(T_{\mathsf{e}} - T_{\mathsf{p}}) + Q_{\mathsf{p}} + \nabla\cdot(\overleftrightarrow{\kappa}_{\mathsf{p}}\nabla T_{\mathsf{p}})
$$

and (e,p) $\rightarrow$ (p,e) $+$ MHD Maxwell Eqs.

$$
\begin{aligned}
\partial_t P + \nabla\cdot\left[(3\mathbf{v} + 2\mathbf{v}_{\mathsf{A}})P\right] - (\mathbf{v}\cdot\nabla)P &= 0 \\
\mathbf{E} + (\mathbf{v}\times\mathbf{B})/c &= \eta\ \mathbf{j}
\end{aligned}
$$

where

$$
\begin{aligned}
P &= P(f,r) = F(f)\ R(r); \quad F(f) = P_0/f \\
\Rightarrow p_{\mathsf{W}} &= \frac{1}{8\pi}\int_{f_0}^{f_{\mathsf{H}}} P(f,r)\ df = \frac{P_0}{8\pi}\ln\left(\frac{f_{\mathsf{H}}}{f_0}\right) R(r) \\
Q_{\mathsf{e}} &= 0 \\
Q_{\mathsf{p}} &= -\frac{P(f_{\mathsf{H}})}{4\pi}\ (\mathbf{v} + \mathbf{v}_{\mathsf{A}})\cdot\nabla f_{\mathsf{H}} \\
(\overleftrightarrow{\kappa}_{\mathsf{e,p}})_{ij} &= \alpha_{ij}(T_{\mathsf{e,p}}) \\
\nu_{\mathsf{ep}} &\propto n\ T_{\mathsf{e}}^{-3/2} \\
\nu_{\mathsf{R}} &\propto 10^{-2} R_{\mathsf{Sun}}\ v_{\mathsf{A}}; \quad \eta = ...?
\end{aligned}
$$