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DYNAMICS OF THE INTERPLANETARY GAS AND MAGNETIC FIELDS*

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ABSTRACT

We consider the dynamical consequences of Biermann's suggestion that gas is often streaming outward in all directions from the sun with velocities of the order of 500–1500 km/sec. These velocities of 500 km/sec and more and the interplanetary densities of 500 ions/cm^3 (10^{14} gm/sec mass loss from the sun) follow from the hydrodynamic equations for a 3×10^6 ° K solar corona. It is suggested that the outward-streaming gas draws out the lines of force of the solar magnetic fields so that near the sun the field is very nearly in a radial direction. Plasma instabilities are expected to result in the thick shell of disordered field (10^{-5} gauss) inclosing the inner solar system, whose presence has already been inferred from cosmic-ray observations.

I. INTRODUCTION

Biermann (1951, 1952, 1957*a*) has pointed out that the observed motions of comet tails would seem to require gas streaming outward from the sun. He suggests that gas is often flowing radially outward in all directions from the sun with velocities ranging from 500 to 1500 km/sec; there is no indication that the gas ever has any inward motion. Biermann infers densities at the orbit of earth ranging from 500 hydrogen atoms/cm^3 on magnetically quiet days to perhaps 10^5/cm^3 during geomagnetic storms (Unsöld and Chapman 1949). The mass loss to the sun is 10^{14}–10^{15} gm/sec. It is the purpose of this paper to explore some of the grosser dynamic consequences of Biermann's conclusions.

For instance, we should like to understand what mechanism at the sun might conceivably be responsible for blowing away the required 10^{14}–10^{15} gm of hydrogen each second, with velocities of the order of 1000 km/sec. All known mechanisms, such as Schlüter's (1954) melon-seed process, are limited more or less to the speed of sound (Parker 1957*b*), minus the deceleration of viscosity and the solar gravitational field. Even at a coronal temperature of 3×10^6 ° K, the thermal velocity of a hydrogen ion is only 260 km/sec, and escape from the solar gravitational field (starting 3×10^5 km above the photosphere) requires 500 km/sec, to say nothing of leaving a residual 500–1000 km/sec at infinity.

Then again, if Biermann's conclusions are correct, we should like to know what configuration of the general solar dipole magnetic field we might expect in interplanetary space. Ionized gas, streaming outward with more or less spherical symmetry from the sun, would be expected to carry the general solar field with it, so that the lines of force are everywhere in the radial direction and extend far out into interplanetary space.

We shall begin our investigation at the sun, which we idealize to be a gravitating ball of mass $M_\odot$ with spherical symmetry. We shall at first completely neglect any solar magnetic fields. With r denoting distance measured from the center of the sun, we shall take the effective surface of the sun (so far as the outward flow of gas is concerned) to be $r = a$ and choose $a = 10^6$ km, representing the outer solar corona. We denote the kinetic temperature of the gas by $T(r)$, its density by $N(r)$, and its radial velocity by $v(r)$. We shall suppose the conditions at $r = a$ to be given, T_0, N_0, v_0. Optical observations suggest (van de Hulst 1953) that N_0 is of the order of 3×10^7/cm^3. The mean value of

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v_0 is not large in the solar corona, the only evidence of outward motion being the existence of streamer-like structures, so conspicuous in eclipse photographs of the solar corona.

Now the total outward flux of kinetic energy transported by hydrogen gas with velocity v and kinetic energy density $\frac{1}{2}NMv^2$ is $I(r) = 2\pi MNv^3r^2$ ergs/sec, where M is the mass of the hydrogen atom. Using $N = 500$ cm^3 and $v = 500$ km/sec at the orbit of earth ($r = 1.5 \times 10^{13}$ cm), we find that $I = 1.5 \times 10^{29}$ ergs/sec. This is to be compared to the usual energy loss (van de Hulst 1953) to the corona (2×10^4 ergs/cm^2 sec due to thermal conduction and 1×10^4 ergs/cm^2 sec due to radiation) amounting to about 3×10^{27} ergs/sec. We see then that the flow of gas out of the solar corona, suggested by Biermann, involves 10^2 times as much energy as heating the static corona.

One may reasonable ask, therefore, whether heating of the corona is not merely a by-product of the immensely more energetic phenomenon of outflowing gas. If we had a scheme for ejecting the gas independently of the coronal temperature, then it would certainly seem more reasonable to adopt such a view. But we do possess plausible mechanisms (Biermann 1948; Schwarzschild 1948; Schatzmann 1949, 1951; Schirmer 1950; Burgers 1951; van de Hulst 1953; Cowling 1956; Piddington 1956) for maintaining high coronal temperatures independently of outflowing gas, and we shall find that the outflow of gas can be made to follow rather simply from coronal heating to million-degree temperatures. On the other hand, we do not know of any mechanism which might result in gas leaving the sun at 1000 km/sec and which does not originate as a consequence of a high coronal temperature. Therefore, we shall for the present adopt the supposition that the basic process is the heating of the coronal gases to $\sim 10^6$ ° K. The outflow of gas we take to be a secondary effect. We shall suppose that the heating is able to supply as much as 1.5×10^{29} ergs/sec. Naturally, such a view will ultimately require a careful re-examination of coronal heating mechanisms, taken up in the following paper.

II. STATIC EQUILIBRIUM

The high temperature of the solar corona suggests that the gas associated with it is fully ionized. Thus the total gas pressure is $2NkT$. For static equilibrium we have the usual barometric relation,

$$0 = \frac{d}{dr}\,(2NkT) + \frac{GM_{\odot}MN}{r^2}. \tag{1}$$

For ionized hydrogen of the densities we are considering, the thermal conductivity is (Chapman 1954)

$$\kappa(T) \cong 5 \times 10^{-7} T^n \text{ ergs/cm}^2 \text{ sec }^\circ\text{K}, \tag{2}$$

where $n = \frac{5}{2}$. For neutral hydrogen, elementary kinetic theory yields $2.5 \times 10^3\ T^n$, where $n = \frac{1}{2}$.

Sufficiently far from the sun, where there are no local heat sources, the steady-state heat-flow equation, $\nabla \cdot [\kappa(T)\nabla T] = 0$, requires that $T(r)$ fall off with distance as $r^{-1/(n+1)}$. If for the moment we suppose that there are no sources of coronal heat beyond $r = a$, then

$$T(r) = T_0 \left(\frac{a}{r}\right)^{1/(n+1)}, \tag{3}$$

and we may immediately integrate equation (1) to give

$$N(r) = N_0 \left(\frac{r}{a}\right)^{1/(n+1)} \exp\left\{\left[\frac{\lambda(n+1)}{n}\right]\left[\left(\frac{a}{r}\right)^{n/(n+1)} - 1\right]\right\}, \tag{4}$$

where λ is the dimensionless parameter $GM_{\odot}M/2kT_0a$. We see that, for $n > 0$, $N(r)$ becomes infinite, according to

$$N(r) \sim N_0 \left(\frac{r}{a}\right)^{1/(n+1)} \exp\left[\frac{-\lambda(n+1)}{n}\right] \tag{5}$$

as r becomes large. For $n < 0$, $N(r)$ vanishes at infinity. For $n = 0$ we have $[(n+1)/n][1 - (a/r)^{n/(n+1)}] \to \ln(a/r)$, so that

$$N(r) = N_0 \left(\frac{a}{r}\right)^{\lambda - 1}. \tag{6}$$

The pressure $p(r) = 2NkT$ varies as

$$p(r) = p_0 \exp\left\{\left[\frac{\lambda(n+1)}{n}\right]\left[\left(\frac{a}{r}\right)^{n/(n+1)} - 1\right]\right\}, \tag{7}$$

when $n \neq 0$, or as

$$p(r) = p_0 \left(\frac{a}{r}\right)^{\lambda} \tag{8}$$

if $n = 0$.

We see, then, that, with the temperature varying as in equation (3) and with n at least as large as the 0.5 for neutral hydrogen, we have non-vanishing pressure at infinity,

$$p(\infty) = p_0 \exp\left[\frac{-\lambda(n+1)}{n}\right] \tag{9}$$

for hydrostatic equilibrium. With $n = 2.5$ (for ionized hydrogen) and $a = 10^6$ km, $T_0 = 1.5 \times 10^{6}\,°$ K, and $M_{\odot} = 2 \times 10^{33}$ gm, we have $\lambda = 5.35$ and $p(\infty) = 0.55 \times 10^{-3}\, p_0$. Even $n = 0.5$ (for un-ionized hydrogen) yields $p(\infty) = 10^{-7}\, p_0$. With standard coronal conditions, $N_0 = 3 \times 10^7/\text{cm}^3$, $T_0 = 1.5 \times 10^{6}\,°$ K, we have $p_0 = 2N_0kT_0 \cong 1.3 \times 10^{-2}$ dynes/cm^2. Hence $p(\infty) = 0.6 \times 10^{-5}$ dynes/cm^2 for $n = 2.5$ and 1.3×10^{-9} for $n = 0.5$.

But at infinity we can expect no more than the interstellar gas pressure, arising from, say, 10 hydrogen atoms/cm^3 at 100° K, or 1.4×10^{-13} dynes/cm^2. Before $p(\infty)$ could be this small, n would have to be as small as 0.27.

Since we know of no general pressure at infinity which could balance the $p(\infty)$ computed from equation (9) with the expected values of n, we conclude that probably it is not possible for the solar corona, or, indeed, perhaps the atmosphere of any star, to be in complete hydrostatic equilibrium out to large distances. We expect always to find some continued outward hydrodynamic expansion of gas—without considering the evaporation from the high-velocity tail of the Maxwellian distribution (Spitzer 1947; van de Hulst 1953).

III. STATIONARY EXPANSION

Having shown that there is no hydrostatic equilibrium solution with vanishing pressure at infinity, we shall now consider what steady expansion we might expect from the solar corona. In particular, we shall be interested to see whether we can obtain Biermann's outflow of gas with not unreasonable coronal temperatures.

We do not know quantitatively in what manner the heating of the solar corona is distributed over r, and in particular we do not know to what distance it extends. We shall, therefore, merely assume that $T(r)$ is given, rather than try to compute it by some general heat-flow equation.

The stationary expansion of the solar corona (which we have just shown is inevitable when the pressure at infinity is negligible) satisfies the equation of motion,

$$NMv\frac{dv}{dr} = -\frac{d}{dr}(2NkT) - GNMM_{\odot}\frac{1}{r^2}, \qquad (10)$$

and the equation of continuity,

$$\frac{d}{dr}(r^2 N v) = 0\,, \qquad (11)$$

if we suppose that the corona possesses spherical symmetry. It follows immediately from equation (11) that

$$N(r)\,v(r) = N_0 v_0 \left(\frac{a}{r}\right)^2. \qquad (12)$$

We shall find it convenient to introduce the dimensionless variables $\xi = r/a$, $\tau = T(r)/T_0$, $\lambda = GMM_{\odot}/2akT_0$, $\psi = \frac{1}{2}Mv^2/kT_0$. Then, using equation (12) to eliminate N, we may reduce equation (10) to

$$\frac{d\psi}{d\xi}\left(1 - \frac{\tau}{\psi}\right) = -2\xi^2 \frac{d}{d\xi}\left(\frac{\tau}{\xi^2}\right) - \frac{2\lambda}{\xi^2}. \qquad (13)$$

To integrate equation (13),[1] obtaining ψ as a function of ξ, let us suppose that the temperature is maintained (by heating mechanisms) at the uniform value T_0 from $r = a$ out to some radius $r = b$. Beyond $r = b$ we suppose that the heating vanishes. Since the outward expansion of the corona consumes 1.5×10^{29} ergs/sec and thermal conduction is not capable of transporting even 1 per cent as much energy, we may reasonably take T to be negligible beyond $r = b$.

Hence, in $r < b$ we have $\tau = 1$, and equation (13) immediately yields

$$\psi - \ln\psi = \psi_0 - \ln\psi_0 + 4\ln\xi - 2\lambda\left(1 - \frac{1}{\xi}\right), \qquad (14)$$

where the constant of integration has been chosen so that $\psi = \psi_0$ at $r = a$. Beyond $r = b$ we have $\tau \simeq 0$ and

$$\psi(\xi) = \psi\left(\frac{b}{a}\right) - \lambda\left(\frac{a}{b} - \frac{1}{\xi}\right). \qquad (15)$$

But we shall find that b is rather larger than a, so that the escape velocity from b is small compared to the 500 or 1000 km/sec which we attain at $r = b$. Thus, to a good approximation, $\psi(b/a)$ represents the gas velocity at large distances from the sun.

Now steady outward flow requires a definite value of v_0 for any given T_0. This manifests itself in the fact that equation (14) does not yield real values of $v(r)$ for all $r > a$ unless v_0 has a particular value. Let

$$Y = 4\ln\xi - 2\lambda\left(1 - \frac{1}{\xi}\right),$$

$$Z = \psi - \ln\psi\,.$$

[1] Note that if we let $q = \ln\tau$ and $F = \psi/\tau - \ln(\psi/\tau)$, so that $\psi/\tau \equiv H(F)$, then we may write eq (13) in the simple form $dF/dq + H(F) = L(q)$, where $L(q) = 1 - 2(d\xi/d\tau)[\xi^2 d(\tau\xi^2)/d\xi + \lambda/\xi^2]$ and is a known function of q.

The value of ψ at $\xi = 1$ is rather less than unity.[2] Thus Y and Z both decrease from their initial values at $\xi = 1$. The quantity Y reaches a minimum[3] at $\xi = \lambda/2$ and thereafter increases monotonically as ξ goes to infinity; Z reaches a minimum at $\psi = 1$ and thereafter increases monotonically. If we wish equation (14) to possess a solution ψ which is real (and positive since $v[r]$ must be real) for all values of $\xi \geq 1$, we see that Y and Z must round their minimum points at the same value of ξ. Thus we must have $\psi = 1$ when $\xi = \lambda/2$, or

$$\psi_0 - \ln \psi_0 = 2\lambda - 3 - 4 \ln \frac{\lambda}{2}. \tag{16}$$

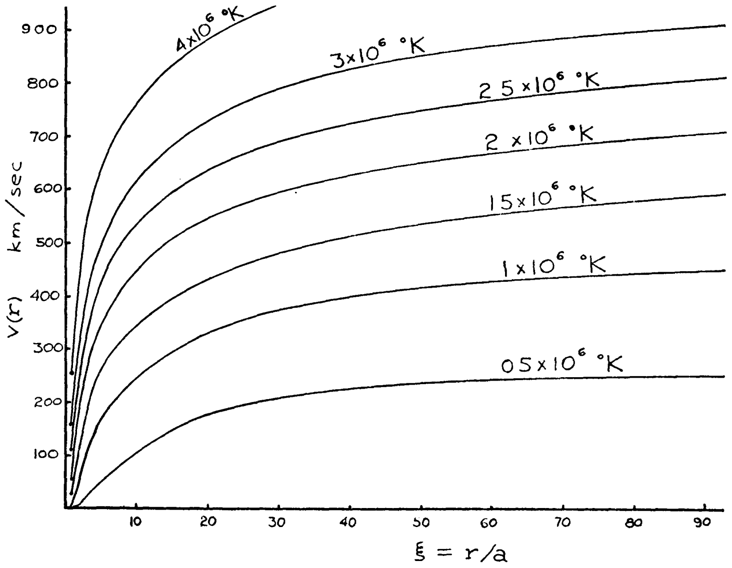

FIG. 1.—Spherically symmetric hydrodynamic expansion velocity $v(r)$ of an isothermal solar corona with temperature T_0 plotted as a function of r/a, where a is the radius of the corona and has been taken to be 10^{11} cm

This yields a value of ψ_0 (or v_0) such that the outward expansion of gas is steady. It follows that

$$\psi - \ln \psi = -3 - 4 \ln \frac{\lambda}{2} + 4 \ln \xi + \frac{2\lambda}{\xi}. \tag{17}$$

If we were by some means suddenly to pump gas out of the sun at a greater rate than the $4\pi a^2 v_0 N_0$ given by equation (16), we would soon achieve a new steady outflow with the same v_0 but with increased N_0, assuming T_0 to remain unchanged.

Figure 1 gives the outward gas velocity $v(r)$ as a function of $\xi = r/a$ for various temperatures T_0. We have put $a = 10^{11}$ cm and $M = 1.66 \times 10^{-24}$ gm, so that $\lambda = 8.0 \times 10^6/T_0$ and $v^2 = 1.7 \times 10^{-2}\, T_0 \psi$ km/sec. We see from Figure 1 that the 500 km/sec required by Biermann's analysis is reached at $r = 5a$ for $T_0 = 3 \times 10^6$ ° K, $36a$ for $T_0 = 1.5 \times 10^6$ ° K, and $200a$ for $T_0 = 1.0 \times 10^6$ ° K. Thus even the 160 km/sec thermal velocities of 10^6 ° K are sufficient to push gas out of the solar gravitational field (escape velocity 500 km/sec) and give the gas an additional 500 km/sec.

[2] We obviously are not interested in the solution for large ψ_0, where the gas starts at $r = a$ with already supersonic velocity.

[3] Provided that $\lambda > 2$.

That this effect is a result of spherical, rather than one-dimensional, expansion may be demonstrated by considering the general equation of continuity in n dimensions:

$$N(r)\,v(r) = N_0 v_0 \left(\frac{a}{r}\right)^{n-1}.$$

We maintain the gravitational form $GM_{\odot}MN/r^2$, and in place of equation (14) we have

$$\psi - \ln\psi = \psi_0 - \ln\psi_0 + 2(n-1)\ln\xi - 2\lambda\left(1 - \frac{1}{\xi}\right). \tag{18}$$

It is the term in $\ln\xi$ which yields the unlimited velocity as $\xi \to \infty$. For the spherical case we have three dimensions, and the coefficient of $\ln\xi$ is 4. Now, for one dimension the $\ln\xi$ term disappears, and the right-hand side decreases monotonically with increasing ξ. Hence $\psi - \ln\psi$ must decrease monotonically with increasing ψ, and it follows that the maximum value of ψ, occurring at $\xi = \infty$, cannot be greater than unity. Thus v is limited to less than the thermal velocity.

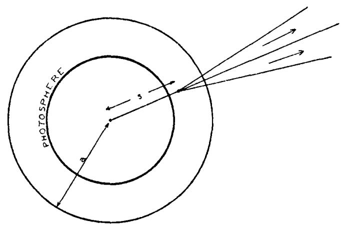

FIG 2.—Spherical expansion from a point at a distance s from the center of the sun, simulating hypothetical outflow of gas from an active region

Suppose that, instead of spherical expansion centered about the sun, we consider a more local spherical expansion centered around some hypothetical active region on the surface of the sun and confined to a narrow cone, as shown in Figure 2. Then we have the continuity condition,

$$N(r)\,v(r) = N_0 v_0 \left(\frac{a-s}{r-s}\right)^2, \tag{19}$$

where s is the distance of the origin of the spherical expansion from the center of the sun. In place of equation (14), we obtain

$$\psi - \ln\psi = \psi_0 - \ln\psi_0 + 4\ln\left[\left(\xi - \frac{s}{a}\right)\Big/\left(1 - \frac{s}{a}\right)\right] - 2\lambda\left(1 - \frac{1}{\xi}\right), \tag{20}$$

assuming the same gravitational force, $-GM_{\odot}MN/r^2$, as before. We find that ψ must equal unity where ξ has the value

$$\xi = \left(\frac{\lambda}{4}\right)\left[1 + \left(1 - \frac{8s}{\lambda a}\right)\right]^{1/2}, \tag{21}$$

which determines the value of v_0 for steady-state flow. We note that we must have $s/a < \lambda/8$: Otherwise there is no solution of equation (20) starting with $\psi < 1$ at $\xi = 1$ and going to $\psi > 1$ at $\xi = \infty$, because there is no minimum of the right-hand side of equation (20) to match the minimum of the left-hand side at $\psi = 1$; thus, as in the one-dimensional case, ψ would be limited to values less than or equal to 1 if $s/a \geq \lambda/8$. Figure 3 shows the outward gas velocity $v(r)$ as a function of r for $T_0 = 2 \times 10^6$ ° K and $s/a = 0.4$ ($\lambda/8 = 0.5$) against the curves at 2 and 2.5×10^6 ° K for $s = 0$. We see that, when $s > 0$, larger outward velocities result from the same temperature.

IV. CORONAL HEATING AND MASS LOSS

Consider the problem of heating an expanding solar corona. As was pointed out earlier, the necessary expansion consumes at least 1.5×10^{29} ergs/sec. If we could assume that 5×10^{-5} of the electromagnetic radiation escaping from the sun was absorbed by the corona, then adequate heat would be supplied, and no further restrictions would need to be imposed. But let us suppose that the corona cannot be heated electromagnetically and so, presumably, must be warmed by mechanical means (van de Hulst 1953). We shall suppose that by some mechanical process, such as acoustical or hydromagnetic waves, energy is transported from the photosphere out through the corona[4] and is finally absorbed in thermal motions beyond $r = a$ to heat the coronal gas. Then a rough upper limit to the mechanical transport is probably given by the product of the thermal energy density, U, and the speed of sound, or thermal velocity, u.

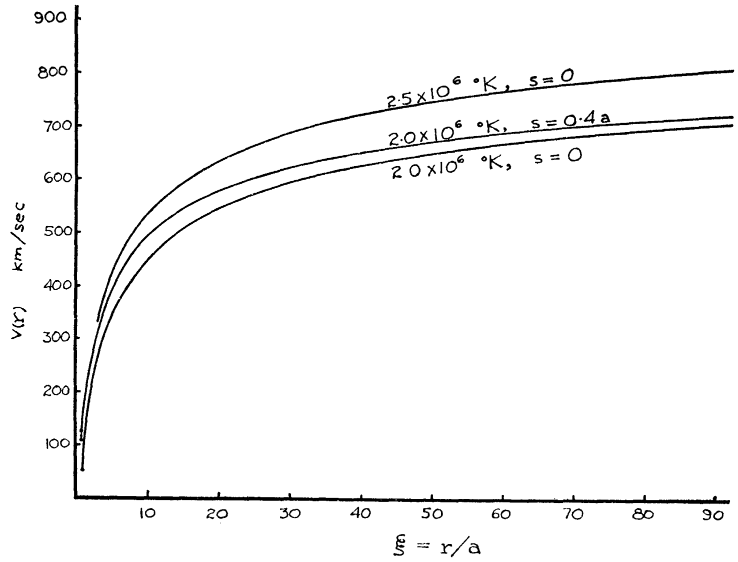

FIG 3—Hydrodynamic expansion velocity outward from a point distant 0 4a from the center of the sun with a coronal temperature of 2×10^{6}° K, compared to expansion with spherical symmetry about the center.

Now if we suppose that the corona is heated only out to a distance $r = b$, where the velocity achieves some value v_m (which we shall later take to be 500 km/sec), then the total transport of kinetic energy at $r = b$ is $(\frac{1}{2}N_m M v_m^3) \times (4\pi b^2)$. At smaller values of r the transport is $(\frac{1}{2}NMv^3) \times (4\pi r^2)$ and is less than the total at $r = b$. The difference must be transported by the coronal heating mechanism, requiring a flux,

$$I(r) = \tfrac{1}{2}M\left[N_m v_m^3 \frac{b^2}{r^2} - N v^3\right] \text{ergs/cm}^2 \text{ sec}.$$

Using equation (12) to eliminate N_m and N, we have

$$I(r) = N_0 v_0 \left(\frac{a}{r}\right)^2 kT_0 (\psi_m - \psi).$$

[4] In the following paper we suggest that it is probably hydromagnetic waves that are responsible for the heating by Fermi acceleration of ions (Fermi 1949, 1953).

Now, if the rate of transport of the thermal energy density $2NkT$ has a characteristic velocity w, then, by the definition of w, the total transport is $2wNkT$, plus the convection $2vNkT$. Neglecting gravitational potential energy, the total must equal $I(r)$, so that, ultimately,

$$w = v\,[\tfrac{1}{2}(\psi_m - \psi) - 1]\,.$$

The ratio of w to the thermal ion velocity $u = (3kT_0/M)^{1/2}$ becomes

$$\frac{w}{u} = (\tfrac{2}{3})^{1/2}\psi^{1/2}\,[\tfrac{1}{2}(\psi_m - \psi) - 1]\,, \tag{22}$$

which is plotted in Figure 4 as a function of r, taking ψ_m to be the value of ψ at which $v = 500$ km/sec. We have suggested that in nature w/u will not greatly exceed unity.

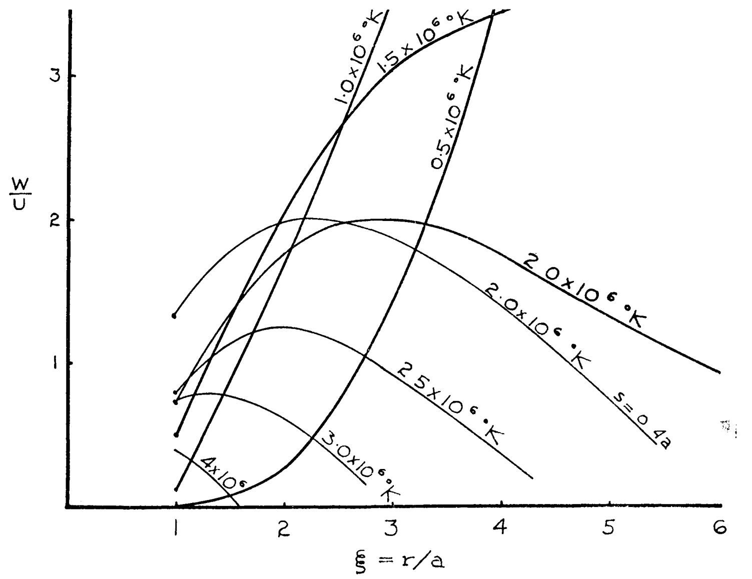

FIG 4.—Ratio of the effective transport velocity to thermal velocity necessary for maintaining the indicated coronal temperatures

Finally, consider the solar mass loss. It is

$$\frac{dM_\odot}{dt} = 4\pi a^2 N_0 M v_0\,. \tag{23}$$

The outward velocity, v_0, and the mass loss are given in Figure 5 for $N_0 = 3 \times 10^7/\text{cm}^3$, taking M equal to the mass of a hydrogen ion. (Note that Biermann's mass loss of 10^{14} gm requires that $v_0 \simeq 160$ km/sec where $N_0 = 3 \times 10^7$.)

Consider Figures 4 and 5 with $T_0 = 3 \times 10^6\,^\circ$ K out to $r = 5a$. It follows that the mass loss is the required 10^{14} gm/sec, the final outward velocity is the required 500 km/sec, and the mechanical transport velocity, w, does not exceed the thermal velocity, u. A coronal temperature of 2 or $3 \times 10^6\,^\circ$ K over an extended region around the sun would seem to be, then, the simplest origin of the outflowing gas suggested by Biermann.

Naturally, our choice of $a = 10^6$ km has been entirely arbitrary, as has been our assumption of spherical symmetry and a uniform temperature[5] for a distance beyond

[5] A better model might be, perhaps, to start with T_0 equal to the observed 1.5×10^6 at $r = a$ and let the temperature increase outward to $3 \times 10^6\,^\circ$ K at, say, $r = 2a$.

$r = a$. We would not expect the actual conditions to be as simple as we have assumed. Unfortunately, our present observational knowledge does not allow construction of a more detailed model, nor, indeed, does it supply much information concerning coronal temperatures several solar radii from the sun. We hope that such information may be forthcoming.

V. GENERAL SOLAR MAGNETIC FIELD

Thus far we have ignored the presence of magnetic fields. Consider the effect of the general outflow of solar gas, suggested by Biermann, upon the general solar dipole magnetic field. Given that there is a general efflux of gas from the sum, the question is whether the gas which flows outward from the sun is threaded by the lines of force of the general solar field. If the gas is not threaded by the general field but flows up through the field from some field-free region beneath the photosphere in streams which force their way between the lines of force, then the outflowing gas probably has no great effect on the solar dipole field. At most, one would expect a few local perturbations of the general field whose time average is very nearly an ordinary magnetic dipole.

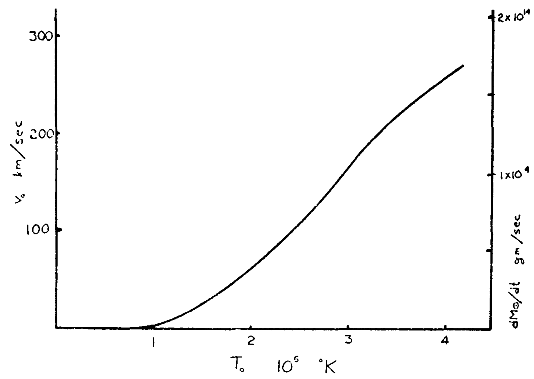

FIG 5 —Steady outward velocity at $a = 10^{11}$ cm and the resulting solar mass loss if the hydrogen density is $3 \times 10^7/\text{cm}^3$, as a function of coronal temperature

But suppose, on the other hand, that there are no field-free regions in the sun from which the gas may issue, so that each cubic meter of gas flowing outward from the sun is threaded by magnetic lines of force from the main bulk of the sun. Then we expect that the outward-streaming gas, because it is ionized, will carry the imbedded lines of force with it. The lines, being imbedded in both the sun and the ejected gas, will be stretched out radially as the gas moves away from the sun. If beyond some distance $r = b$ from the sun the steady efflux of gas has some semblance of spherical symmetry, then after a time the lines of force will be entirely radial (neglecting the rotation of the sun). The radial configuration will be as universal as Biermann's outward-gas motion, which is responsible for it.

One does not observe field-free regions on the surface of the sun (Babcock and Babcock 1955) either near the poles or in the more chaotic equatorial regions. The hydrodynamic convection in the ionization zone beneath the photosphere apparently mixes the gas and whatever fields it carries, from the observed surface of the photospheric granules down to a depth of 10^5 km or more; magnetic fields are observed at the photosphere and hence may be assumed down to 1 or 2×10^5 km. The steady dipole character of the north and south polar magnetic regions suggests that their fields may extend all the way through the sun. Thus there is no observational reason to believe that there are field-free regions anywhere in the sun.

What is more, it has been shown elsewhere (Parker 1957*a*, *b*) that a blob of field-free gas encysted in a large-scale magnetic field may spread out without limit along the lines of force and will also tend to split lengthwise. Thus at least one dimension of the blob will decrease without limit, and the gas will soon diffuse into the surrounding magnetic field. Therefore, we tentatively suggest that the gas flowing out from the sun is not field-free but carries with it magnetic lines of force originating in the sun. Hence, with the more or less steady outflow suggested by Biermann, we expect a radial solar magnetic field, falling off approximately as $1/r^2$ in interplanetary space.

It is a simple matter to compute the steady-state magnetic field resulting from a spherically symmetric outflow of gas from a rotating star. We shall suppose that beyond some distance $r = b$ both the solar gravitation and outward acceleration by high coronal temperature may be neglected, so that the outward velocity is a constant, v_m; in the frame of reference rotating with the sun we have the gas velocity, in spherical co-ordinates,

$$v_r = v_m\ , \qquad v_\theta = 0\ , \qquad v_\phi = \omega\,(r - b)\sin\,\theta\ , \tag{24}$$

where ω is the angular velocity of the sun.

The streamline with azimuth ϕ_0 at $r = b$ is given by

$$\frac{r}{b} - 1 - \ln\left(\frac{r}{b}\right) = \frac{v_m}{b\omega}(\phi - \phi_0)\,. \tag{25}$$

Since one end of each line of force is fixed in the sun and we are considering only steady-state conditions, it is obvious that the lines of force of the magnetic field coincide with the streamlines. Since $\nabla \cdot \boldsymbol{B} = 0$, it follows[6] that the magnetic field at the point (r, θ, ϕ) is given by

$$\begin{aligned} B_r(r,\ \theta,\ \phi) &= B(\theta,\ \phi_0)\left(\frac{b}{r}\right)^2, \\ B_\theta(r,\ \theta,\ \phi) &= 0\ , \\ B_\phi(r,\ \theta,\ \phi) &= B(\theta,\ \phi_0)\left(\frac{\omega}{v_m}\right)(r - b)\left(\frac{b}{r}\right)^2 \sin\,\theta\,, \end{aligned} \tag{26}$$

where ϕ and ϕ_0 are related by equation (25).

$B(\theta, \phi_0)$ represents the field at $r = b$. If we believe that only the solar dipole field threads the escaping gas, then $B(\theta, \phi_0) = B_0 \cos\theta$. If, however, the more complex fields of the equatorial regions participate, then $B(\theta, \phi_0)$ is of a more complicated nature, and we might expand B as $\Sigma A_{nm} P_n^m (\cos\theta) \cos m\,(\phi_0 - \phi_m)$. But in either case, the lines of force follow equation (25), which is sketched in Figure 6 for $b = 5 \times 10^{11}$ cm, $v_m =$ 1000 km/sec.

It is obvious from equation (25) that the lines of force spiral more and more with increasing r. For small r, $B_\phi \cong 0$, but, for large r, $B_\phi/B_r \sim (\omega r \sin\theta)/v_m$, which increases without limit. The surface on which B_ϕ is equal to B_r and on which each line of force makes an angle $\pi/4$ with the radius vector is

$$r - b = \frac{v_m}{\omega}\sin\,\theta\,.$$

Or, since $v_m \gg \omega b$, it is the circular cylinder,

$$r \cong \frac{v_m}{\omega}\sin\,\theta\,. \tag{27}$$

[6] To prove this formally, we need only note that if no net flux is being transported out of the sun, then we must have $\boldsymbol{B} = \alpha \boldsymbol{v}$, where α is a scalar function. Since $\nabla \cdot \boldsymbol{B} = 0$, we obtain a partial differential equation for α, the characteristic curves of which are (25), and yielding $\alpha = (a/r)^2$ along all such curves.

The angular velocity of the sun is approximately $\omega \simeq 2.7 \times 10^{-6}$. Thus the radius of the cylinder on which B_ϕ becomes equal to B_r is 2.5 a.u. for $v_m = 1000$ km/sec.

VI. INTERPLANETARY MAGNETIC FIELD AND RETARDATION OF SOLAR ROTATION

In the solar corona the 1-gauss solar dipole field has an energy density which is of the same order as the kinetic energy density and hydrostatic pressure of the coronal gas. However, as may be seen from the equation of continuity (12) and from equation (24), the kinetic energy density $\frac{1}{2}NMv^2$ falls off with increasing r ($> b$) only as r^{-2}, whereas equation (26) indicates that the magnetic energy density will decrease as r^{-4} so long as r is less than the value given by equation (27), and as r^{-2} thereafter. It follows, therefore, that we do not expect the general solar dipole field significantly to influence the motion of the outflowing gas, once the gas has left the solar corona. Whether or not the denser fields associated with active regions in the equatorial zone are ever inflated to such an extent that they contribute to Biermann's general streaming of gas is an open question. Obviously, very much more energy would be required to extend such fields because the magnetic energy could be enormously larger than the final kinetic energy. If the magnetic fields were sufficiently dense, one might expect significant devia-

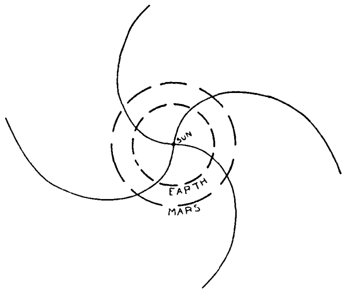

FIG 6.—Projection onto the solar equatorial plane of the lines of force of any solar field which is carried to infinity by outward-streaming gas with velocity 10^3 km/sec

tions from the velocity field given in equation (24), with a tendency toward rigid rotation with the sun. As Biermann (1957*b*) has pointed out, the motions of comet tails give no indication of any rigid rotation. Therefore, we shall suppose that the velocity field (24) does not lead to an unreasonable picture, equation (26), of the solar magnetic field in interplanetary space.

Consider the torque which might be exerted on the sun by the spiral field given in equation (26). The torque about the z-axis exerted across the surface of a sphere of radius r by the stresses in the magnetic field $\boldsymbol{B}$ is

$$L = r^2 \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi \left(\frac{B_r B_\phi}{4\pi}\right) r \sin\theta$$
$$= \frac{b^2}{4\pi}\frac{\omega}{v_m}\left(1-\frac{b}{r}\right)\int_0^\pi d\theta \sin^3\theta \int_0^{2\pi} d\phi_0 B^2(\theta, \phi_0)\,.$$

For the case of a simple dipole field at the sun with a density B_0 at the poles, we have $B = B_0 \cos\theta$ and

$$L(r) = \tfrac{2}{15} b^4 \frac{\omega}{v_m} B_0^2 \left(1-\frac{b}{r}\right).$$

$L(r)$ vanishes at $r = b$ because of the idealized velocity field adopted, completely ignoring the magnetic stresses. The maximum torque which we could conceivably expect to be exerted on the sun is $L(\infty)$. With $v_m = 1000$ km/sec and $b = 2 \times 10^{11}$ cm, we obtain $L(\infty) = 5.8 \times 10^{30}$ dynes/cm. The moment of inertia, I, of the sun is of the order of 2×10^{54} gm/cm², so that the characteristic angular deceleration time is $I\omega/L = 10^{18}$ seconds or 3×10^{10} years. We conclude, therefore, that the torque exerted on the sun by our interplanetary model of the solar field is not serious.

VII. PLASMA INSTABILITY AND THE INTERPLANETARY MAGNETIC SHELL

It was shown in Section IV that within 1 or 2 a.u. of the sun we expect the lines of force of the general solar magnetic field to extend radially outward, so that the field density falls off as r^{-2}. Taking the gas from the sun to be moving with more or less constant velocity in a radial direction, once it gets a little way out from its source at the corona, we see that expansion of the gas is primarily perpendicular to $\boldsymbol{r}$, and hence to $\boldsymbol{B}$, within 1 or 2 a.u. from the sun. As result of the low collision frequency of the ions in the tenuous gas, the thermal motions will become anisotropic, with the gas pressure p_n perpendicular to $\boldsymbol{B}$ rather less than p_s parallel. It is readily shown from the equations of motion of a plasma (Parker 1957*b*, 1958*b*) that the velocity of propagation of a hydromagnetic wave is $[B/(4\pi\rho)^{1/2}]\,[1 + 4\pi(p_n - p_s)/B^2]^{1/2}$ when the thermal motions are not isotropic. When $4\pi(p_s - p_n) > B^2$, this velocity becomes purely imaginary, and the wave does not propagate. Instead, its amplitude increases exponentially with time. It is estimated that a wave length of 10^6 km has a characteristic time of growth of the order of 4 hours; shorter wave lengths grow correspondingly faster.

It follows, therefore, that the smooth idealized fields such as we have sketched in equation (25) and in Figure 6 are certainly to be complicated by a disorganized region extending from about 1 a.u. outward;[7] the inner solar system is surrounded by a thick shell of tangled magnetic field of about 10^{-5} gauss. Observational evidence of both the radial field inside 1 a.u. and the disorganized shell surrounding 1 a.u. comes from the onset and subsequent decay of cosmic rays from solar flares (Meyer, Parker, and Simpson 1956; Simpson 1957).

The dynamic instability leading to the thick shell of disorganized field is discussed at greater length elsewhere (Parker 1958*a*, *b*). We have mentioned it here as a warning to the reader against taking too literally any of the smooth idealized models which we have constructed in this paper.

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[7] Beyond 2 5 a u., where the lines of force near the equatorial plane may be largely azimuthal, we would expect that perhaps $p_n > p_s$, in which case there is a second instability that may occur (Parker 1958*b*) to disorganize the field further.

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