Fully kinetic simulation of ion acoustic and dust-ion acoustic waves

S. M. Hosseini Jenab,[1,2,a)] I. Kourakis,[2,b)] and H. Abbasi[1,c)]
[1]*Amirkabir University of Technology, 424 Hafez Ave., Tehran, Iran*
[2]*Center for Plasma Physics, Department of Physics and Astronomy, Queen's University Belfast, BT7 1NN Northern Ireland, United Kingdom*

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A series of numerical simulations is presented, based on a recurrence-free Vlasov kinetic model using kinetic phase point trajectories. All plasma components are modeled kinetically via a Vlasov evolution equation, then coupled through Poisson's equation. The dynamics of ion acoustic waves in an electron-ion and in a dusty (electron-ion-dust) plasma configuration are investigated, focusing on wave decay due to Landau damping and, in particular, on the parametric dependence of the damping rate on the dust concentration and on the electron-to-ion temperature ratio. In the absence of dust, the occurrence of damping was observed, as expected, and its dependence to the relative magnitude of the electron vs ion temperature(s) was investigated. When present, the dust component influences the charge balance, enabling dust-ion acoustic waves to survive Landau damping even in the extreme regime where $T_e \simeq T_i$. The Landau damping rate is shown to be minimized for a strong dust concentration or/and for a high value of the electron-to-ion temperature ratio. Our results confirm earlier theoretical considerations and contribute to the interpretation of experimental observations of dust-ion acoustic wave characteristics. © *2011 American Institute of Physics*. [doi:10.1063/1.3609814]

I. INTRODUCTION

Ion acoustic (IA) waves (IAWs) constitute a fundamental mode in ion dynamics, in which inertial ions oscillate dynamically against a background of electrons, the latter providing the necessary restoring force.[1] One of the first plasma modes has been studied theoretically;[2] IAWs and their properties have been investigated experimentally in various experiments.[3] One of the fundamental characteristics of their dynamics is Landau damping, which describes wave decay occurring when the wave's phase speed lies in the vicinity of the ion thermal speed. It can be shown that Landau damping is dominant when the ion temperature is near the electron temperature, viz. $T_i \simeq T_e$, while it is minimized for $T_e \gg T_i$, due to the associated modification of the phase speed.[4] Landau damping is a fundamental element in the kinetic description of plasma waves and is nonetheless absent in the fluid plasma description.[1]

Plasmas in Space and in laboratory are often characterized by the presence of massive charged particles (dust).[5–8] In the presence of the dust component in the background, ion-acoustic waves give rise to the so-called dust-ion acoustic (DIA) mode.[9]The frequency ω of this wave lies in the range $kv_{Td} \ll kv_{Ti} \ll \omega \ll kv_{Te}$ (where v_{Ts} denotes the thermal velocity of species $s =$ dust, ions, or electrons, respectively). In this ionic frequency range, electrons may be considered as inertialess, while the massive dust particles (much heavier than ions) will remain stationary. The restoring force comes from the electron pressure and the inertia is provided by the ions. Importantly, the dust concentration affects the charge balance and, thus, modifies the phase speed of DIA waves, resulting in weaker Landau damping in all cases (and even in the range near $T_i \simeq T_e$).[4] A simple way to explain this effect is by considering the phase speed of ion-acoustic waves:

$$v_{ph}^2 = \frac{n_{i0}}{n_{e0}} \frac{k_B T_e}{m_i} \frac{1}{1 + k^2 \lambda_{De}^2} + 3 \frac{k_B T_i}{m_i}, \tag{1}$$

where T_e and T_i denote the electron and ion temperature, respectively, and m_i is the ion mass (k_B is the Boltzmann constant). The ion-to-electron density ratio at equilibrium (n_{i0}/n_{e0}) equals unity in the absence of dust. However, in the presence of negatively charged dust particles, the ratio increases as $n_{i0}/n_{e0} = 1 + Z_d n_d/n_{e0}$ due to electron depletion (because part of the electrons now reside on the dust grains), leading to an increase in the phase speed; consequently, the number of "resonant" particles in the vicinity of the phase speed is reduced, and the Landau damping rate is minimized. These theoretically predicted properties of DIA waves have been established experimentally.[10,11]

The kinetic-theoretical approach to plasma modeling has led to various numerical algorithms introduced based on plasma statistical-mechanical principles. Following the seminal work of Vlasov[12] and relying on the same theoretical principles, a number of numerical schemes were proposed for the kinetic description of plasma dynamics; worth mentioning is the early scheme proposed by Cheng and Knorr,[13] who introduced a "splitting method" for Vlasov simulations. Regretfully, their numerical algorithm introduces a flaw, namely *the recurrence effect*: initial states of the distribution

a)Electronic mail: jenab@aut.ac.ir.
b)Electronic addresses: IoannisKourakisSci@gmail.com and i.kourakis@qub.ac.uk.
c)Electronic mail: abbasi@aut.ac.ir.

function will reappear in the simulation periodically (in around recurrence time $t_r = 2\pi/(kdv)$ or its multiples, where k is the wavenumber and dv is the grid step in the velocity direction), an effect that is entirely numerical and reflects no physical truth, yet make the results of the simulation unreliable beyond time t_r. This problem is inherited by recent numerical integration methods.[14–16] We shall briefly discuss below how this nuisance is overcome in our numerical study.[17]

In this article, we have undertaken extensive numerical simulations of the dynamics of ion-acoustic waves, in the presence of a massive dust component. We have adopted a fully kinetic approach in which the dynamics of all plasma components (namely, electrons, ions, and dust) is followed during the simulation via a Vlasov description. This is a highly demanding computational algorithm, which ensures improved accuracy in the description of the plasma dynamics and is free from simplifying assumptions (e.g., thermal electrons or static dust). As a first step in our approach, we have considered the linear propagation of IA and DIA waves, focusing on tracing the Landau damping effect on these modes in various regimes, and in particular considering the effect(s) of the electron-to-ion temperature ratio and of the dust concentration. We have employed a numerical method based on following phase point trajectories, along an idea proposed in Ref. 18. Applying a characteristics method to the Vlasov equation, the latter is reduced to two first order differential equations, allowing for the phase point trajectories to be followed in phase space. The main advantage of this method is that the initial value of distribution function of phase points remains untouched throughout the simulation process, so these points may move through the mesh points used at different steps through the calculation. In this way, unlike other methods, phase points are not restricted to coincide with mesh points. This separation between phase points and mesh points is a novel element in the algorithm, which enables us to overcome the recurrence effect. Detailed analysis shows that the recurrence effect can be eliminated via a randomized allocation of phase points in the velocity direction, at the initial stage of the simulation.[17]

The layout of this study goes as follows. In Sec. II, we introduce the basic aspects of our theoretical model. In Sec. III, the results of the simulation for different parameters are presented and discussed. Section IV dedicated to a summary of the presented work.

II. MODEL AND NUMERICAL PROCEDURE

We consider a three-component plasma, consisting of electrons (mass m_e, charge $q_e = -e$), singly ionized ions (mass m_i, charge $q_i = +e$), and negatively charged dust (mass m_d, charge $q_d = -Z_d e$). For ion-acoustic waves, the dust component below will be "switched off." The one-dimensional (1D) Vlasov-Poisson system of equations will be adopted in our study. Each plasma species is described by a Vlasov equation:

$$\frac{\partial f_s}{\partial t} + v\frac{\partial f_s}{\partial x} + \frac{q_s E(x,t)}{m_s}\frac{\partial f_s}{\partial v} = 0, \quad s = i, e, d, \tag{2}$$

where the variable v denotes velocity in phase space (i.e., v_x in a 1D description).

The densities of the plasma components are given upon integration as

$$n_s = n_{s0}\int f_s dv \quad (\text{where}\, s = d, i, e) \tag{3}$$

and are coupled through Poisson's equation

$$\frac{\partial^2\phi}{\partial x^2} = \frac{e}{\epsilon_0}(n_e - n_i + Z_d n_d). \tag{4}$$

For our purpose, we shall rescale the above system by normalizing space by λ_{Di}, time by ω_{pi}^{-1}, where $\omega_{pi} = [n_{i0}e^2/(m_i\epsilon_0)]^{1/2}$ denotes the ion plasma frequency and $\lambda_{Di} = [\epsilon_0 k_B T_i)/(n_{i0}e^2)]^{(1/2)}$ is the characteristic ion Debye length. The velocity variable v has been scaled by the ion thermal speed $v_{th_i} = (k_B T_i/m_i)^{1/2}$, while the electric field and the electric potential have been scaled by $k_B T_i/(e\lambda_{Di})$ and $k_B T_i/e$, respectively (here k_B is Boltzmann's constant). The densities of the three species are normalized by n_{i0}. We define the dust parameter as the ratio $\delta = \frac{Z_d n_{d0}}{n_{i0}}$ (which vanishes in the absence of dust) and the normalized electron density $n = \frac{n_{e0}}{n_{i0}}$. Note that the quasi-neutrality condition

$$n_{e0} - n_{i0} + Z_d n_{d0} = 0 \tag{5}$$

is assumed to hold at the initial step in our simulation, implying the relation:

$$n + \delta - 1 = 0. \tag{6}$$

The reduced Vlasov system of equations now reads

$$\frac{\partial f_e}{\partial t} + v\frac{\partial f_e}{\partial x} - \frac{m_i}{m_e}E(x,t)\frac{\partial f_e}{\partial v} = 0, \tag{7}$$

$$\frac{\partial f_i}{\partial t} + v\frac{\partial f_i}{\partial x} + E(x,t)\frac{\partial f_i}{\partial v} = 0, \tag{8}$$

and

$$\frac{\partial f_d}{\partial t} + v\frac{\partial f_d}{\partial x} - Z_d\frac{m_i}{m_d}E(x,t)\frac{\partial f_d}{\partial v} = 0, \tag{9}$$

$$\frac{\partial^2\phi}{\partial x^2} = \tilde{n}_e + \tilde{n}_d - \tilde{n}_i. \tag{10}$$

The normalized density functions read: $\tilde{n}_e = n\int f_e dv$, $\tilde{n}_i = \int f_i dv$, $\tilde{n}_d = \delta\int f_d dv$. Once the distribution functions are obtained from the above kinetic equations, a numerical integration provides the densities of different species. Using those densities in the Poisson equation, the electric potential (and the field) can be obtained. The electric field is input in the Vlasov equations, and the next step densities are thus obtained. This circle is iterated, and the results are retained at every step, testing to confirm that energy is preserved throughout the procedure.

For the sake of completeness and for later reference, it is appropriate to add here that the dispersion relation (1) (for the phase speed $v_{ph} = \omega/k$) becomes, for our purposes (adopting the scaling above),

$$\tilde{\omega}^2 = \tilde{k}^2 \left(\frac{\theta}{1 - \delta + \theta \tilde{k}^2} + 3 \right). \tag{11}$$

A Maxwellian state at equilibrium is assumed for all species. In order to excite ion acoustic waves, a small periodic perturbation was added to the initial condition for the ions, viz, at time $t = 0$

$$\tilde{n}_e = n \int \left(\frac{1}{2\pi}\right)^{1/2} \left(\frac{m_e}{m_i}\frac{T_i}{T_e}\right)^{1/2} \exp^{-\left(\frac{m_e}{m_i}\frac{T_i}{T_e}\right)\frac{v^2}{2}} dv, \tag{12}$$

$$\tilde{n}_i = \int \left(\frac{1}{2\pi}\right)^{1/2} \exp^{-v^2/2}(1 + \alpha \cos x) dv, \tag{13}$$

and

$$\tilde{n}_d = \delta \int \left(\frac{1}{2\pi}\right)^{1/2} \left(\frac{m_d}{m_i}\frac{T_i}{T_d}\right)^{1/2} \exp^{-\left(\frac{m_d}{m_i}\frac{T_i}{T_d}\right)\frac{v^2}{2}} dv \tag{14}$$

in which α is the amplitude of disturbance imposed on ions.

For comparison, in order to excite electron plasma (Langmuir) waves, we would impose a small perturbation on the electron distribution function, while leaving the other two species in a Maxwellian state, initially.

We have undertaken a $1+1$ dimensional simulation in which a phase space grid $(N_x, N_{vx}) = (25, 10\,000)$ is introduced by considering at the initial step a resolution of 4 phase points per mesh cell. In order to avoid the aforementioned recurrence effect, we have used random sampling of phase points over phase space at the initial step of the simulation. The other parameters in the simulation were $m_d/m_i = 10^5$, $m_i/m_e = 10^2$, $T_i/T_d = 400$, $Z_d = 1000$, time step $dt = 0.01$, $\alpha = 0.01$, and $L = \lambda = 5\pi$, where L is the length of the simulation box and λ is the wavelength. For the sake of simplicity, we shall set $\theta = T_e/T_i$ in the following.

III. RESULTS AND DISCUSSION

A. Langmuir waves excited by ionic perturbations

We have started our study by considering an isothermal electron-ion plasma (for $\delta = 0$), assuming $\theta = 1$. In this case, significant Landau damping is expected to dominate over IA wave propagation, essentially preventing any ion vibration. This was confirmed by our numerical result, shown in Fig. 1(a). However, due to the periodic perturbation on the ion distribution, the electrons start to oscillate at their (plasma) eigenfrequency, so a Langmuir type wave is spontaneously excited in the plasma. In support of this interpretation, we have carried out a separate simulation for comparison, in which a Langmuir oscillation was purposefully excited (via a periodic perturbation over the electrons; see [Fig. 1(b)]). Both results are depicted in Fig. 1(c), where we see that the frequencies of the two periodic oscillations coincide. We conclude that Landau damping prevents ion-acoustic oscillations in the isothermal case $T_e = T_i$, as expected, while Langmuir waves may be excited by ionic perturbations of the initial state. A similar qualitative behavior is witnessed for higher θ, up to (roughly) 2.5 (see discussion below).

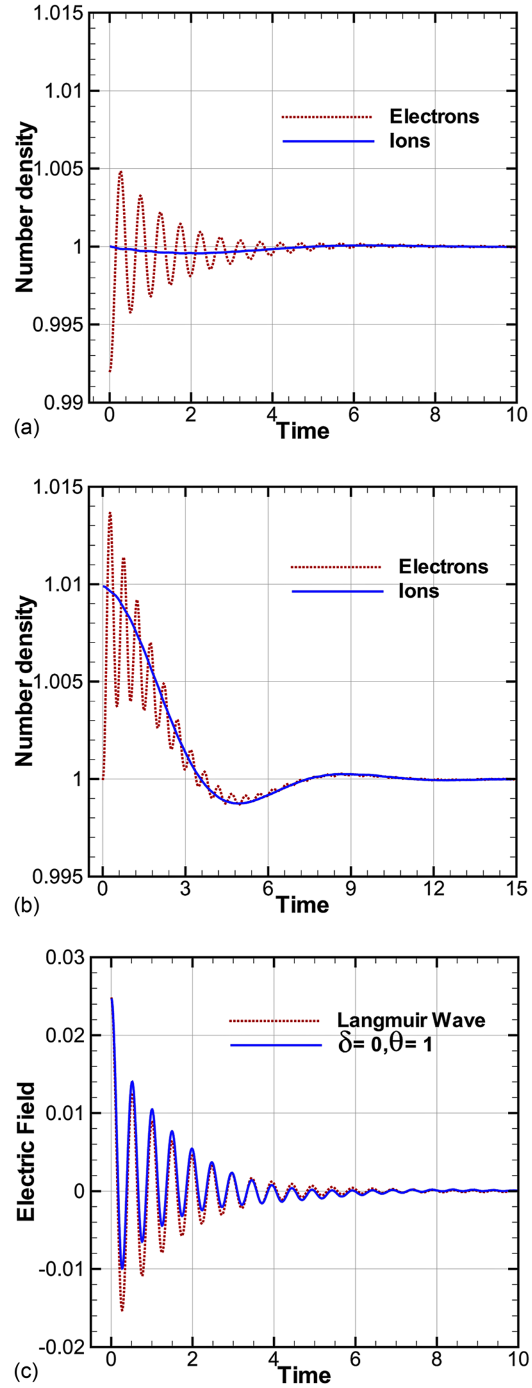

FIG. 1. (Color online) (a) The number density of electrons and ions is depicted versus time (scaled by ω_{pi}^{-1}), for $\delta = 0$ and $\theta = 1$. Here, a small initial perturbation is imposed on ions. (b) The number density of electrons and ions is depicted versus time, by imposing a small initial perturbation on the electron distribution function. The Langmuir mode is the only mode properly excited here (apart from a very weak ion-acoustic oscillation which is quickly damped). (c) Comparison between Langmuir waves (red dashed curve), excited by an initial perturbation of the electron distribution, and electron oscillations obtained for $\delta = 0$, $\theta = 1$ (blue solid curve). The (reduced) electric field is depicted versus time. The frequencies of the two oscillations coincide.

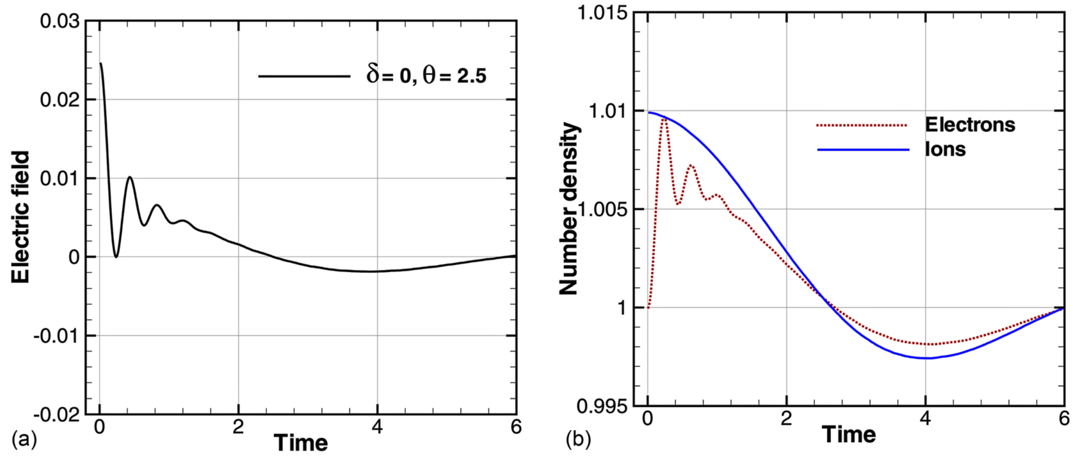

FIG. 2. (Color online) (a) Electric field versus time, for $\delta = 0$ and $\theta = 2.5$. A superposition of Langmuir and damped ion acoustic oscillations is observed. (b) Number density of electrons and ions for $\delta = 0$ and $\theta = 2.5$.

B. Ion acoustic waves

In this part, we focus on the role of the electron-to-ion temperature ratio θ on ion-acoustic waves. We saw above that an initial disturbance in the ionic distribution function is rapidly damped out for $\theta = 1$, yet excites short-lived electron plasma oscillations in the plasma. We have observed a similar behavior for θ higher than (but near) one, and up to 2.5, approximately, when ion-acoustic excitations are sustained for longer times. We have depicted our results for $\theta = 2.5$ (and $\delta = 0$) in Fig. 2. A rapid Langmuir oscillation appears to be dominant in the first two ion-periods, approximately, and then adapts itself by responding to the ion-acoustic oscillations, which are dominant in both species above $t \gtrsim 2$.

In Fig. 3, the parametric variation of the Landau damping rate of IAWs over a broad range of θ values is computed. As anticipated, increasing θ results in reduced Landau damping rate especially in the interval of (2.5, 100). The results converges to a constant for $T_e \gg T_i$.

The convergence of our simulation algorithm has been tested at every run, by following the evolution of the total energy versus time. The result is depicted in Fig. 4, where we see that energy was conserved within 0.2% at all times.

C. Dust ion-acoustic waves

Effect of dust concentration on dust-ion acoustic waves propagation. We have investigated the parametric effect of the dust-to-ion charge density ratio (δ) on the frequency and Landau damping rate of dust-ion acoustic waves (DIAWs). The dust presence affects the phase velocity of the DIAWs. An increase in the dust density results in an increase in the phase velocity, which entails a decrease in the Landau damping rate. In our simulations, the addition of dust appears to enable IAW propagation, by minimizing Landau damping. This is true even in the "isothermal" regime where $T_e = T_i$ ($\theta = 1$), in contrast to the e-i plasma case, in which Landau damping would be significant in this case. In our simulation, a continuous change in dust density from $\delta = 0$ to $\delta = 1.0$

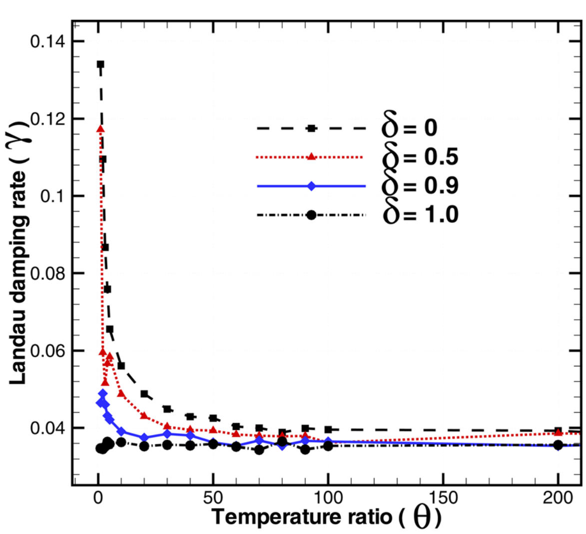

FIG. 3. (Color online) The Landau damping rate of IA and DIA waves (γ) is computed versus the temperature ratio $\theta = \frac{T_e}{T_i}$. The damping rate is found to be lower (slower decay), the higher the value of θ or/and the higher the value of $\delta = \frac{Z_d n_{d0}}{n_{i0}}$ (for a higher dust concentration).

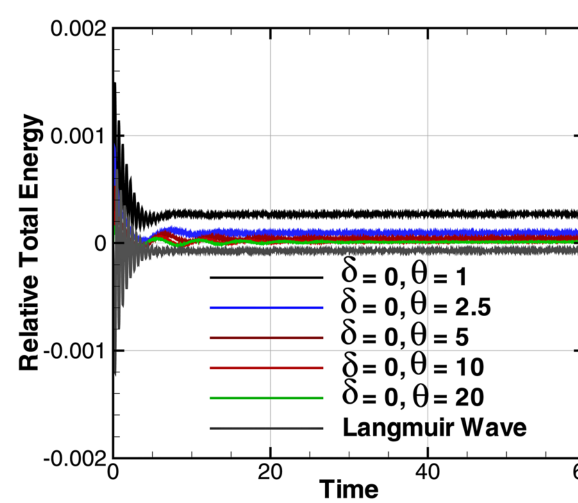

FIG. 4. (Color online) The relative total energy variation $\Delta(t) = \frac{A(t)-A(0)}{A(0)}$ is computed from numerical data. In all our simulations, the relative total energy variation remained under 0.2ArXiv and the total energy was conserved, as expected theoretically. (The data in the inset label tag the curves top to bottom: all of the curves remain close to zero, at all times.)

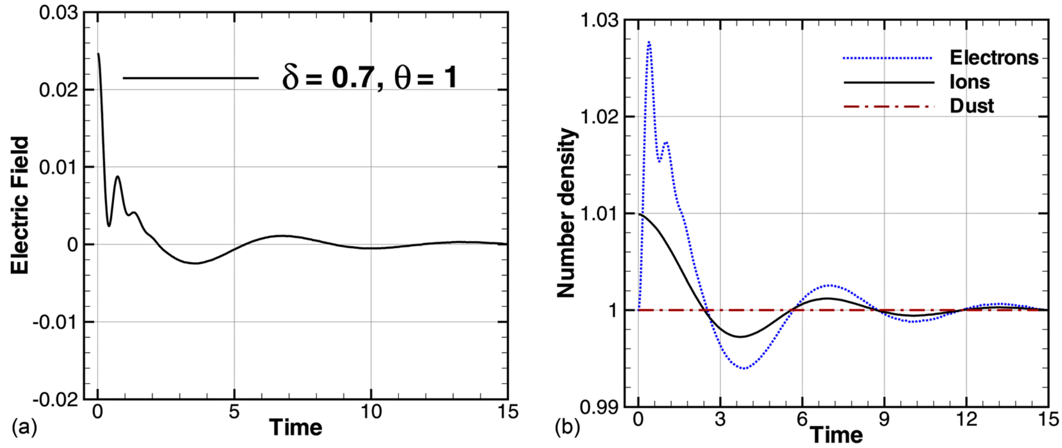

FIG. 5. (Color online) (a) The electric field and (b) the number densities are depicted versus time, for a dust-laden isothermal plasma (taking $\delta = 0.7$, $\theta = 1$). All quantities are scaled (reduced), as explained in the text. In the right panel (b), the dashed/continuous/dot-dashed curves correspond to electrons/ions/dust, respectively.

seems to result in Langmuir waves being suppressed, while on the other hand, dust-ion acoustic waves are generated, as clearly visible in Fig. 5 (for charge density ratio $\delta = 0.7$). A superposition of the two modes (electron plasma waves and DIAWs) is excited in the case of $\delta = 0.7$ and $\theta = 1$. In the beginning (for $t < 2$ approximately), fast Langmuir-type vibrations of the electric field and the electron number density are clearly visible on the plots (see Fig. 5). At higher time ($t > 2$), the slower DIAWs dominate over Langmuir oscillations, as clearly visible in the electric field and ion number density plots in Fig. 5.

Effect of dust concentration on the Landau damping of dust-ion acoustic waves. As expected theoretically, and also earlier confirmed experimentally,[10,11] increasing the dust density results in a decrease in Landau damping rate (see Fig. 6(b)), which may be attributed to an increase in the wave frequency and consequently phase speed (see Fig. 6(a)).

In Figs. 6(a), 7, and 8, we notice that the value of the frequency observed (measured numerically) slightly exceeds the theoretical value, as it is calculated from dispersion relation (11). This is clearly due to the fact that both electron inertia and dust thermal effects are neglected in (11).

IV. CONCLUSIONS

We have carried out extensive numerical simulations of plasma dynamics on the ion-acoustic scale, both in an electron-ion and in a dusty plasma. A fully kinetic code was developed for this purpose, treating all plasma components kinetically, based on Vlasov equation, then coupling their dynamics through Poisson's equation. A novel algorithm was employed in order to rule out the notorious recurrence effect.[17]

The propagation of ion-acoustic waves is known to be suppressed by Landau damping if the phase speed lies in the vicinity of the thermal speed of ions and this effect is in fact maximum for higher ionic temperature, in the region $T_e \leq T_i$. In this case, the excitation of Langmuir waves is inevitable due to the small electron mass, so that electrons can easily respond to an initial perturbation, even if it is imposed on the ions. For $T_e > T_i$, the phase velocity of IAWs will be

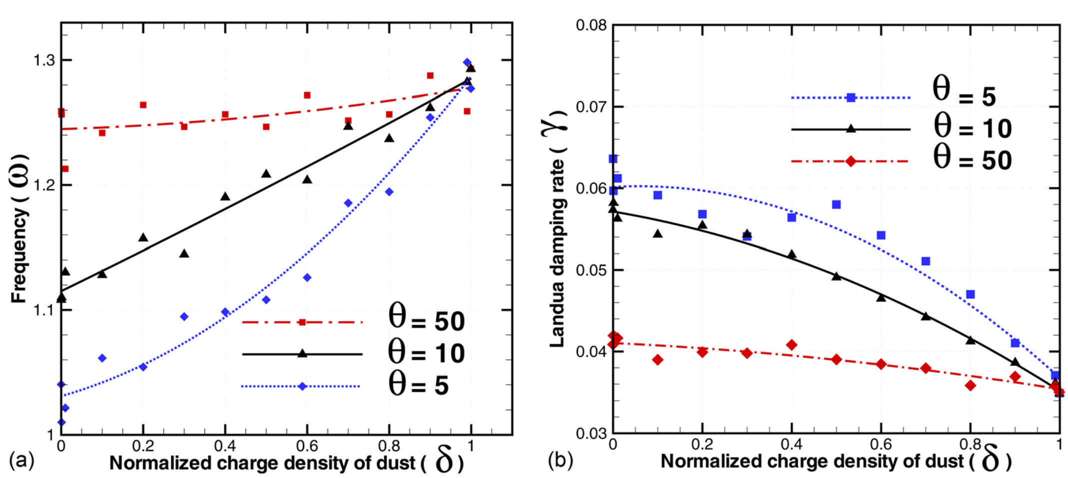

FIG. 6. (Color online) (a) The (normalized) oscillation frequency (ω/ω_{pi}) and (b) the Landau damping rate (γ) are depicted versus the (normalized) dust to ion charge density ratio ($\delta = \frac{Z_d n_{d0}}{n_{i0}}$). Higher values of δ lead to an increase in frequency and to a decrease in damping rate.

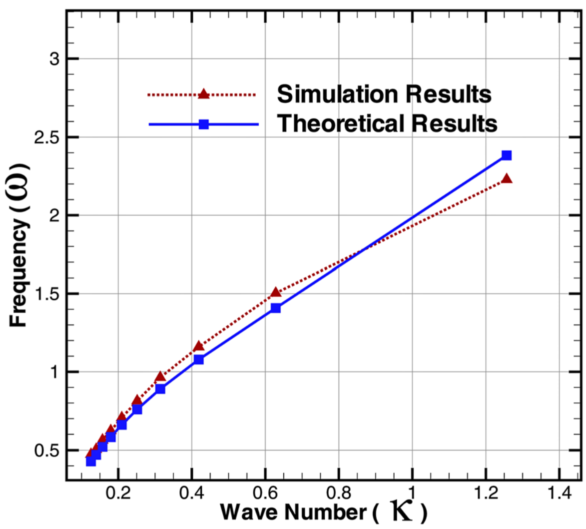

FIG. 7. (Color online) The (reduced) oscillation frequency (ω/ω_{pi}) versus the reduced wavenumber ($k\lambda_{Di}$). Solid line shows the theoretical values based on the relation 11, as dotted lines presents simulations results, both depicted for $\delta = 0.5$, $\theta = 5$.

much higher than the thermal velocity of ions, and consequently, the Landau damping rate decreases and IAWs can propagate through the plasma. We have investigated the Landau damping rate over a broad range of values of θ (temperature ratio), finding that it manifests a decreasing trend (lower damping for higher ion temperature) and approximately converges for $\theta > 100$.

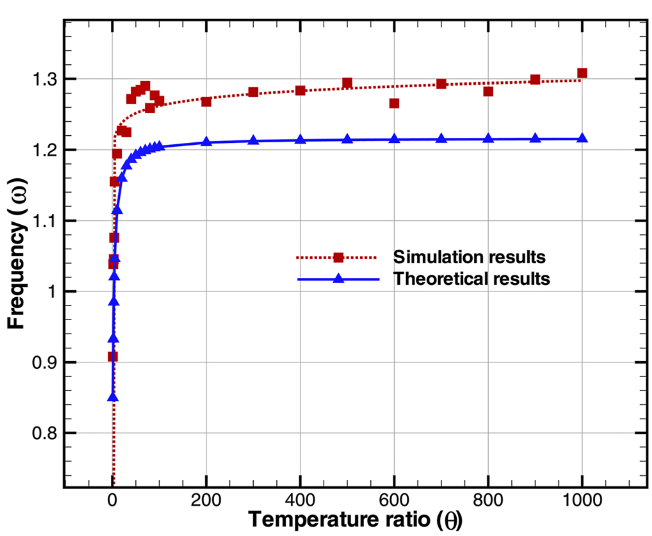

FIG. 8. (Color online) The reduced DIAW frequency (ω/ω_{pi}) is depicted for $k = 0.4$, $\delta = 0.5$. Here, the solid line shows the theoretical value based on relation (11), while the dotted line represents the simulation results. A difference of approximately 5% between the two curves can be attributed to the approximations underlying (Eq. (11)) (electron inertia and dust thermal effect neglected).

Considering the presence of a massive charge component in a dusty plasma, we have shown that the dust presence leads to a modification of the phase speed of dust-ion-acoustic waves, thus resulting in weaker Landau-type wave damping. We have investigated the parametric dependence of the Landau damping phenomenon on the ion temperature (considering even the extreme case when $T_i = T_e$) and on the dust concentration, to find that higher values of T_e/T_i or/and higher dust concentration entail a reduction in damping rate.

Our results corroborate earlier theoretical considerations and contribute to the interpretation of experimental observations of dust-ion acoustic wave characteristics.

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