Nonlinear modulated electrostatic wavepackets in e-p-i plasmas or pair-ion plasmas doped with a stationary charged component

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Abstract

The nonlinear amplitude modulation of electrostatic wave packets propagating in a three-component plasma is investigated, by employing a two-fluid plasma description. Focus is made on electron-positron-ion (e-p-i) plasmas; alternatively, the model describes pair-ion (eg. fullerene) plasmas contaminated by a uniform and stationary minority charged particle species (e.g. defects, or dust particulates). Wave propagation parallel to the external magnetic field is considered.

1. Introduction. *Pair plasmas* (p.p.), i.e. plasmas consisting of equal mass and opposite charge sign particles, feature properties which do not exist in ordinary (e-i) plasmas. For instance, since the positively and negatively charged particles in p.p. respond on the same frequency scale (unlike electrons and heavy ions), ion-acoustic waves have no counterpart in electron-positron (e-p) plasmas, where the electrostatic (ES) wave dispersion may be of high-frequency parabolic (Langmuir-like) type [1-3], and neither does Faraday rotation. Recently, the production of pair fullerene-ion plasmas in laboratory [3] has enabled experimental studies of pair plasmas rid of intrinsic problems involved in electron-positron plasmas, namely pair recombination processes and strong Landau damping.

In real, e.g. astrophysical contexts, e-p plasmas may be enriched by the additional presence of positive ions. Electron-positron-ion (e-p-i) plasmas appear in the early universe, in active galactic nuclei (AGN) and in pulsar magnetospheres, and may also be created in laboratory (see Refs. in: [4]). Weakly nonlinear low-frequency ES modes in e-p-i plasmas were considered in [5]. Here, we investigate high-frequency oscillations of (light) electrons and positrons (or pair ions) against a neutralizing background of (heavier) ions which, given the frequency range of interest, may be considered immobile.

2. The model. We consider a collisionless plasma, consisting of *two* inertial species, say 1 and 2, of opposite charge $q_{1/2} = s_{1/2}Zq$ (here $s_1 = -s_2 = +1$) and equal mass $m_{1/2} = m$, and a fixed background of heavier ions (mass m_i, charge $q_i = +Z_i e$); e is the (absolute) electron charge. In specific, 1 and 2 may represent electrons and positrons, in e-p-i plasmas, or heavier C_{60}^{+} ions in a pair fullerene-ion plasma, where a minority ion species (e.g. defects) is present.

The (two) inertial dust fluids are described by the moment evolution equations

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial (n_\alpha u_\alpha)}{\partial x} = 0 \, , \qquad \frac{\partial u_\alpha}{\partial t} + u_\alpha \frac{\partial u_\alpha}{\partial x} = -s_\alpha \frac{q}{m}\frac{\partial \phi}{\partial x} - \frac{1}{m n_\alpha}\frac{\partial p_\alpha}{\partial x} \, , \tag{1}$$

where t and x are time and (1D) space variables and n_α, v_α and p_α denote the density, velocity and pressure, respectively, of species $\alpha = 1, 2 \equiv +, -$ (of charge sign $s_\alpha = \pm 1$). The equation of state $p_\alpha = \gamma n_\alpha k_B T_\alpha$ is assumed to hold, along with $p_\alpha = C n_\alpha^\gamma$; the specific heat ratio $\gamma = (f+2)/f$ (for f degrees of freedom) is here equal to 3; here, T_α is the temperature of species α; k_B is Boltzmann's constant.

The electric potential ϕ obeys Poisson's equation

$$\frac{\partial^2 \phi}{\partial z^2} = -4\pi \sum_{s(pecies)=1}^{3} n_s q_s = 4\pi e [Z(n_2 - n_1) - Z_i n_i] \,, \tag{2}$$

where $n_{1(+)}$ and $n_{2(-)}$ denote the positron (or positive ion) and electron (or negative ion) density, respectively; the background ion density $n_i = n_{i,0}$ is constant. The rhs in Eq. (2) cancels at equilibrium, due to the quasi-neutrality condition $Z(n_{2,0} - n_{1,0}) - Z_i n_{i,0} = 0$.

3. Perturbative analysis. The system of (5) Eqs. (1, 2) for the state vector $\mathbf{S} = \{n_1, u_1; n_2, u_2; \phi\}$ supports harmonic electrostatic waves in the form $\mathbf{S} = \hat{\mathbf{S}} \exp[i(k\mathbf{r} - \omega t)] + \text{c.c.}$ In order to study the variation (modulation) of the amplitude(s) $\hat{S}_j$ (here $j = 1, ..., 5$) due to nonlinearity, we consider small deviations from the equilibrium state $\mathbf{S}^{(0)} = (n_{1,0}, 0; n_{1,0}, 0; 0)^T$, viz. $\mathbf{S} = \mathbf{S}^{(0)} + \epsilon \mathbf{S}^{(1)} + \epsilon^2 \mathbf{S}^{(2)} + ...$, where $\epsilon \ll 1$ is a smallness parameter. We assume $Sj^{(n)} = \sum_{l=-n}^{n} S_{j,l}^{(n)}(X, T)\, e^{il(k\mathbf{r} - \omega t)}$, for all 5 state variables ($S_{j,-l}^{(n)} = S_{j,l}^{(n)*}$, for reality), allowing the amplitude(s) to depend on the stretched (*slow*) coordinates $X = \epsilon(x - v_g\, t)$ and $T = \epsilon^2\, t$; here $v_g = \omega'(k)$ is the wave's group velocity.

The calculation, particularly lengthy yet straightforward, can be found in [4]; also see [6] for details on the method, which essentially implements the generic reductive perturbation method [7] for ES plasma waves.

The (dominant) first harmonic amplitudes are determined (to order $\sim \epsilon^1$) as

$$n_{+,1}^{(1)} = \frac{n_{-,0} \beta c_s^2 k^2}{\omega^2 - 3\sigma c_s^2 \beta^2 k^2} \frac{Ze\phi_1^{(1)}}{k_B T_-} = \frac{\beta k}{\omega} u_{+,1}^{(1)} \,, \qquad n_{-,1}^{(1)} = -\frac{n_{-,0} c_s^2 k^2}{\omega^2 - 3 c_s^2 k^2} \frac{Ze\phi_1^{(1)}}{k_B T_-} = \frac{k}{\omega} u_{-,1}^{(1)} \,,$$

e.g. in terms of the potential correction $\phi_1^{(1)} \equiv \psi$. We have defined the density ratio $\beta = n_{+,0}/n_{-,0}$, the sound speed $c_s = (k_B T_-/m)^{1/2}$, the temperature ratio $\sigma = T_+/T_-$, and the defect- (background ion) density ratio $\delta = n_3/n_2 \equiv n_{i,0}/n_{-,0}$; see that quasi-neutrality imposes $\beta = 1 - \delta Z_i/Z$, implying $n_+ < n_-$ for positive background ions, i.e. $Z_i > 0$, as implied here (the inverse would hold for $Z_i < 0$); $\beta = 1$ ($\delta = 0$) in p.p. [2].

The linear dispersion relation obtained in $\sim \epsilon^1$ takes the form of a bi-quadratic polynomial equation for ω. Two distinct real solutions are obtained for ω, which for small wave number k values behave as

$$\omega_1 \approx \pm c_{0,L} k \ , \qquad \omega_2 \approx \pm(\omega_g^2 + c_{0,U}^2 k^2)^{1/2} \,, \tag{3}$$

where we defined the characteristic speeds $c_{0,L} = c_s[3\beta(1+\sigma\beta)/(1+\beta)]^{1/2}$ and $c_{0,U} = c_s[3(1+\sigma\beta^3)/(1+\beta)]^{1/2}$ and the gap (cutoff) frequency $\omega_g = \omega_{p,-}(1+\beta)^{1/2}$. The L(ower) curve ω_1 is an acoustic branch, while the U(pper) curve ω_2 determines a Langmuir-like optic mode. These results generalize the known dispersion relation for ES modes in pair plasma [1, 3] (here recovered for $\beta = \sigma = 1$). A numerical investigation shows that increasing the fixed ion species density (i.e. decreasing β) results in lower frequency in

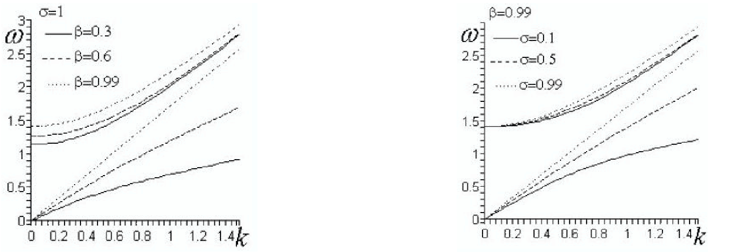


Figure 1: Dispersion relation ω vs. k: effect of variation of β (left) and σ (right).

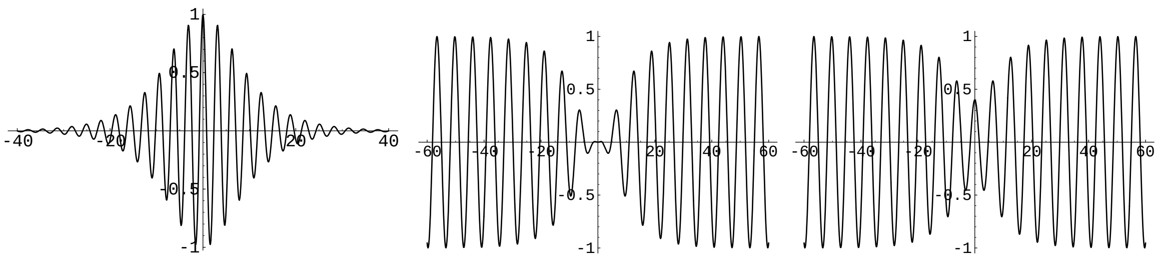


Figure 2: (left) Bright-, (middle) black-, and (right) grey-type soliton solution of (4).

both modes (and, in fact, lower values of $c_{0,U/L}$ and ω_g). On the other hand, for fixed β, decreasing σ results in lower $c_{0,U/L}$ (but does not affect ω_g); see Fig. 1.

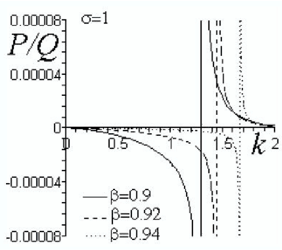
The amplitudes of the 2nd and 0th (constant) harmonic corrections, $S_{j,2}^{(2)}$ and $S_{j,0}^{(2)}$, are obtained in order $\sim \epsilon^2$; the lengthy expressions are omitted for brevity.

4. Nonlinear amplitude evolution equation. In order ϵ^3, a compatibility condition is obtained, in the form of a *nonlinear Schrödinger–type equation* (NLSE)

$$i\,\frac{\partial \psi}{\partial T} + P\,\frac{\partial^2 \psi}{\partial X^2} + Q\,|\psi|^2\,\psi = 0\,, \tag{4}$$

for the potential correction ψ. Both the *dispersion coefficient* $P = \omega''(k)/2$ and the *nonlinearity coefficient* Q, due to carrier wave self-interaction, are lengthy functions of k, σ and β, omitted here, for brevity; the exact expressions can be found in [4]. It may be interesting to trace the behavior of P and Q for long wavelengths, i.e. for $k \ll \lambda_D$ $(= c_s/\omega_{p,-})$. For the lower branch, the coefficients behave as $P \sim -k$ and $Q \sim 1/k$, ensuring modulational stability (since $PQ < 0$: see in §5 below). For the upper branch, $P(k=0)$ =cst., while $Q \sim k^2$: the optic-type upper mode is therefore unstable.

5. (In)stability profile & envelope localized excitations. The perturbed electric potential is $\psi = \epsilon\psi_0\,\cos(kx - \omega t + \Theta) + \mathcal{O}(\epsilon^2)$. It is known [8] that the evolution of a modulated wave whose amplitude obeys Eq. (4) depends on the coefficient product PQ. Eq. (4) supports the plane wave solution $\psi = \psi_0 \exp(iQ|\psi_0|^2 T)$; now, perturbing the amplitude as: $\hat\psi = \hat\psi_0 + \epsilon\,\hat\psi_{1,0}\cos(\tilde k X - \tilde\omega T)$, one obtains the dispersion relation: $\tilde\omega^2 = P\,\tilde k^2\,(P\tilde k^2 - 2Q|\hat\psi_{1,0}|^2)$. If $PQ > 0$, the amplitude ψ is *unstable* for $\tilde k < \sqrt{2Q/P}|\hat\psi_{1,0}|$. If $PQ < 0$, the amplitude ψ will be *stable* to external perturbations.



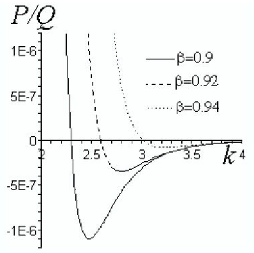


Figure 3: The ratio P/Q vs. k: lower (acoustic) mode β (left); upper mode (right).

This *modulational instability* mechanism is a well known energy localization mechanism in nonlinear dispersive media.

Different localized envelope solutions of Eq. (4) (*envelope solitons*) exist; see in [6] for a brief outline and analytical expressions (also Refs. therein for details). For $PQ > 0$, *bright* envelope modulated wavepackets occur, i.e. localized envelope *pulses* confining the carrier (see Fig. 2a). For $PQ < 0$, dark (*dark* (Fig. 2b) or *grey* (Fig. 2c) envelope solitons exist, modelling a localized envelope *hole* (a *void*) amidst a uniform region.

6. Numerical analysis - conclusions. A numerical analysis shows that both modes are sensitive to variations of the positive-to-negative-ion (or positron-to-electron) density and temperature ratios, β and σ. The lower (acoustic) mode is stable for large wavelengths, and may propagate as a dark-type envelope soliton (a potential dip, or a void). On the other hand, the upper (optic) one is modulationally unstable, and favors the formation of bright-type envelope solitons (pulses). This behavior is depicted in Figs. 3.

These results may be of relevance in experimental [3] and astrophysical [9] contexts. In specific, one may anticipate that the existence of a third minority species in pair plasmas (e.g. defects, or dust) may be used to "tune" the stability of electrostatic modes.

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