Propagation regimes for an electromagnetic beam in magnetized plasma

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The propagation of a Gaussian electromagnetic beam along the direction of magnetic field in a plasma is investigated. The extraordinary (E_x+iE_y) mode is explicitly considered in the analysis, although the results for the ordinary mode can be obtained upon replacing the electron cyclotron frequency ω_c by $-\omega_c$. The propagating beam electric field is coupled to the surrounding plasma via the dielectric tensor, taking into account the existence of a stationary magnetic field. Both collisionless and collisional cases are considered, separately. Adopting an established methodological framework for beam propagation in unmagnetized plasmas, we extend to magnetized plasmas by considering the beam profile for points below the critical curve in the beam-power versus beam-width plane, and by employing a relationship among electron concentration and electron temperature, provided by kinetic theory (rather than phenomenology). It is shown that, for points lying above the critical curve in the beam-power versus beam-width plane, the beam experiences oscillatory convergence (self-focusing), while for points between the critical curve and divider curve, the beam undergoes oscillatory divergence and for points on and below the divider curve the beam suffers a steady divergence. For typical values of parameters, numerical results are presented and discussed. © *2008 American Institute of Physics*.
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I. INTRODUCTION

The phenomenon of self-focusing of electromagnetic beams in nonlinear and dispersive media is of utmost significance in laser plasma interaction[1–14] and in the understanding of nonlinear phenomena, associated with the propagation of high irradiance radio waves in the ionosphere (see, e.g., Refs. 15–17). These studies have focused on unmagnetized (isotropic) plasmas. However, in many situations of interest, the plasma displays anisotropic dielectric behavior due to the presence of a magnetic field. However, self-focusing of electromagnetic beams in magnetoplasmas (plasmas embedded in an external magnetic field) has received relatively less attention. Our investigations aim at partially filling this gap.

An expression for the dependence of the electron density on the irradiance in a collisionless magnetoplasma on account of the ponderomotive force has been derived by Sodha *et al.* in Ref. 18 and was used to obtain the radial dependence of the dielectric tensor and thereby to investigate the self-focusing of an electromagnetic beam. A phenomenological theory was employed therein to investigate Ohmic heating of electrons by the beam, resulting to a radial distribution of electrons (and hence the dielectric tensor) and finally self-focusing of the beam. Chen *et al.*[19] also discussed the nonlinearity associated with an intense electromagnetic wave in collisionless magnetoplasma. The self-focusing in a magnetoplasma was analyzed in Ref. 20, in which the radial redistribution of the electrons is determined by radially outward thermal conduction. Relying on Boltzmann's transfer equation, Tewari and Kumar[21] derived an expression for the dependence of the electron temperature on the irradiance, which is identical to that derived via phenomenological theory, for a wave frequency much higher than the collision frequency; however, that work[18] relied on phenomenological theory to obtain the dependence of electron density on irradiance from the electron temperature dependence. Analyses, corresponding to Ohmic heating of electrons, as the dominant nonlinear mechanism and similar to that in Ref. 18, have been reported by Litvak[22] and Tewari *et al.*[23] Mention should also be made of the application of moment theory to self-focusing in magnetoplasma in Ref. 24 and the prediction of anomalous penetration of laser radiation in overdense magnetoplasma by Sodha *et al.*[25] Recently, the focusing/defocusing of a single Gaussian electromagnetic beam as well as set of interacting coaxial Gaussian electromagnetic beams, propagating along the Earth's magnetic field in the ionosphere, were investigated in Ref. 26 by using the paraxial approximation. That study also accounted for the growth of a sinusoidal instability due to self-focusing.

In a general manner, analyses of beam self-focusing are restricted by the approximation of quadratic nonlinearity of the dielectric tensor and hence miss out on the oscillatory wave guide behavior (see Ref. 27, for instance). Furthermore, even when saturating nonlinearity is considered, the investigation is limited to obtaining a critical curve between the beam radius and the beam power, such that for all points on the critical curve, the beam propagates in a uniform wave guide mode, without convergence and divergence. It has been shown that for points above the critical curve the beam

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undergoes oscillatory convergence (or self-focusing); i.e., the beam width oscillates between the original value and a minimum value as the beam propagates in the plasma. Nothing was mentioned about the behavior of the beam corresponding to points below the critical curve.

In this paper we consider the propagation characteristics of a Gaussian electromagnetic beam propagating along the direction of a static magnetic field. The theoretical framework is presented in Sec. II. The collisionless and collisional plasma cases are respectively considered in Secs. III and IV, wherein the explicit dependence of the beam profile on relevant parameters is investigated. The beam-power versus beam-width space is divided in three regimes, characterized by beam focusing, defocusing, and steady divergence. We define the divider curve in the beam-power versus beam-width plane, such that for all points between the critical and divider curves (defined above) the beam displays oscillatory divergence, while for all points on or below the divider curve the beam undergoes steady divergence. The kinetic theoretical formulation is employed to obtain the dependence of the electron density on the irradiance via the electron temperature. Numerical results for the dependence of self-focusing on the plasma-to-beam and cyclotron-to-beam frequency ratios ω_p/ω, ω_c/ω, on the beam power and on the beam radius will be presented and discussed. Our results are finally summarized and discussed in the concluding Sec. V.

II. THE MODEL

We shall adopt the formalism by Sodha *et al.*[3] to analyze the propagation of a Gaussian electromagnetic beam through a homogeneous plasma, along the direction of an embedded static magnetic field $\vec{B}_0$; the plasma can be treated as neutral everywhere to a very good approximation. The components of the electric vector $\vec{E}$ satisfy the wave equation

$$\left(\sum_j \frac{\partial^2}{\partial x_j}\right)E_i - \frac{\partial}{\partial x_i}\sum_j \frac{\partial E_j}{\partial x_j} + \frac{\omega^2}{c^2}\sum_j \epsilon_{ij}E_j = 0,$$
$$i,j = x,y,z \equiv x_1,x_2,x_3. \tag{1}$$

Here, ω is the beam electric field frequency and c is the speed of light. The dielectric function, a second rank tensor ϵ_{ij}, acquires a simple form if the magnetic field is directed along one of the Cartesian axes, say, z. In this case ϵ_{ij} has only three independent nonvanishing components; namely, $\epsilon_{xx}=\epsilon_{yy}$, $\epsilon_{xy}=-\epsilon_{yx}$, and ϵ_{zz}. The wave equations for $i=x$ and y are coupled, implying that the fields E_x and E_y cannot propagate independently. However, their combinations $E_\pm = E_x \pm iE_y$ can be shown to evolve independently and correspond to the extraordinary (plus sign) and ordinary (minus sign) modes.

The analysis for the propagation of each of the two is similar and hence only essential steps for the extraordinary (X-) mode will be presented; the results for the ordinary (O-) mode can be obtained by substituting $-\omega_c$ for the cyclotron frequency $\omega_c = eB_0/m\,c$ (e is the electron charge). In the first-order approximation (neglecting the product of nonlinear parts of ϵ_+ and ϵ_{zz} with second derivatives of E_+), Eq. (1) reduces to

$$\frac{\partial^2 E_+}{\partial z^2} + \frac{1}{2}\left(1 + \frac{\epsilon_{+0}}{\epsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E_+ + \frac{\omega^2}{c^2}\epsilon_+ E_+ = 0 \tag{2}$$

for $E_+ = E_x + iE_y$, where $\epsilon_+(E) = \epsilon_{xx}(E) - i\epsilon_{xy}(E)$.

It can be shown from phenomenological theory or kinetic theory (e.g., Ref. 28) that when the electron collision frequency ν is much smaller than the wave frequency ω,

$$\epsilon_+ = 1 - (\omega_p^2/\omega^2)(1-\Omega_c)^{-1}, \tag{3a}$$

$$\epsilon_{zz} = 1 - (\omega_p^2/\omega^2), \tag{3b}$$

$$\epsilon_{+0} = 1 - [\Omega_p^2/(1-\Omega_c)], \tag{3c}$$

$$\epsilon_{0zz} = 1 - \Omega_p^2, \tag{3d}$$

where we have defined the ratios

$$\Omega_p = \omega_{p0}/\omega \tag{3e}$$

and

$$\Omega_c = \omega_c/\omega. \tag{3f}$$

Here, ω_{p0} is the plasma frequency in the absence of the em field. Note the presence of the magnetic field via the cyclotron frequency ω_c. In general, wherever appropriate below, the subscript "0" refers to undisturbed plasma quantities (in the absence of the beam, i.e., for $E=0$).

To take into account the cylindrical symmetry of the beam, it is appropriate to transform Eq. (2) into cylindrical coordinates (with azimuthal symmetry). We obtain

$$(1/\delta)\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \epsilon(r,z)\frac{\omega^2}{c^2}E = 0, \tag{4}$$

where

$$\delta = \frac{1}{2}\left[1 + \left(\frac{\epsilon_{+0}}{\epsilon_{0zz}}\right)\right] = \frac{1}{2}\left[1 + \frac{(1-\Omega_c-\Omega_p^2)}{(1-\Omega_c)(1-\Omega_p^2)}\right] \tag{5}$$

[using Eqs. (3c) and (3d)] and

$$\epsilon(r,z) = \epsilon_+/\delta. \tag{6}$$

It is convenient to cast $\epsilon(r,z)$ in the form

$$\epsilon(r,z) = \epsilon_0(z) + \epsilon_1(r,z). \tag{7}$$

We shall henceforth assume that $\epsilon_1 \ll \epsilon_0$ (to be justified *a posteriori*).

To proceed further one may substitute in Eq. (2) for $E(r,z)$ the form

$$E_+(r,z) = A(r,z)\exp i\left(\omega t - \int_0^z k(z)dz\right), \tag{8}$$

where the wave number k is

$$k(z) = (\omega/c)\sqrt{\epsilon_0(z)}. \tag{9}$$

One thus obtains

$$-\frac{2ik(z')}{\delta}\frac{\partial A}{\partial z'}+\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}+\frac{\omega^2}{c^2}\epsilon_1(r,z')A=0. \quad (10)$$

In writing Eq. (10), a term $\partial^2 A/\partial z'^2$ has been neglected (this truncation is legitimate when the fractional change in dielectric function within one wavelength is much less than unity) and the space coordinate was scaled as

$$z'=z/\delta.$$

In the investigation of self-focusing of inhomogeneous beams, it is convenient to introduce the concept of eikonal $S(r,z')$ by defining

$$A=A_0(r,z')\exp[-ik(z')S(r,z')], \quad (11)$$

where A_0, k, and S are real. The eikonal $S(r,z')$ may be expressed[1–3] as

$$S(r,z')=\tfrac{1}{2}r^2\beta(z')+\phi(z'), \quad (12)$$

where $\beta(z')=(1/f)df/dz'$ represents the curvature of the wave front, multiplied by δ, and f is a trial function to be determined (a function of z'). Finally, $\phi(z')$ is a real phase.

Substituting for A from Eq. (11) and for S from Eq. (12) in Eq. (10) and equating the real and imaginary parts on both sides of the equation, one obtains a set of two equations. For an initially Gaussian beam

$$A_0^2(z'=0)=E_{00}^2\exp(-r^2/r_0^2f^2), \quad (13)$$

these two equations lead, in the paraxial approximation, to

$$E_+E_+^*=A_0^2(r,z')=[E_{00}^2/f(z')^2]\exp[-r^2/r_0^2f(z')^2] \quad (14)$$

and

$$\epsilon_0(f)\frac{d^2f}{d\xi^2}=\frac{1}{f^3}\left[1+\rho_0^2f^2\frac{r_0^2f^2}{r^2}\epsilon_1(r,f)\right], \quad (15)$$

where

$$\xi=z'c/r_0^2\omega=cz/\delta r_0^2\omega \quad \text{and} \quad \rho_0=r_0\omega/c.$$

Note that $\epsilon_0(z')=\epsilon_0(f)$ and $\epsilon_1(r,z')=\epsilon_1(r,f)$, since z' is a function of f.

For an initially plane wave, the boundary conditions on Eq. (15) are taken, at $\xi=0$, as

$$df/d\xi=0 \quad \text{and} \quad f=1 \quad (16)$$

to make Eqs. (13) and (14) consistent.

Equation (15) can be numerically integrated using the initial conditions (16) to give f as a function of ξ when functional forms of $\epsilon_0(f)$ and $\epsilon_1(r,f)$ are known. Results for collisionless and collisional plasmas with specific expressions for ϵ are presented in the following sections.

III. COLLISIONLESS PLASMA

The dielectric function $\epsilon(r,f)$ of Eq. (2) corresponding to a ponderomotive nonlinearity for the extraordinary mode is (see Sodha *et al.*[3])

$$\epsilon(r,z)=(1/\delta)[1-\gamma\exp(-\alpha_0E_+E_+^*)], \quad (17)$$

where

$$\gamma=\Omega_p^2/(1-\Omega_c), \quad (18)$$

$$\alpha_0=\frac{3m}{16M}\frac{(2-\Omega_c)}{(1-\Omega_c)^2}\alpha, \quad (19)$$

and

$$\alpha=\frac{e^2M}{6k_0T_0\omega^2m^2}. \quad (20)$$

We denote by m the mass of the electron, M the mass of ion/neutral atom, T_0 the temperature in absence of the beam, and k_0 is Boltzmann's constant. Substituting for $E_+E_+^*$ from Eq. (14) in Eq. (17) and retaining terms up to r^2 (paraxial approximation), one obtains

$$\begin{aligned}\epsilon_0(f)&=(1/\delta)[1-\gamma\exp(-p)],\\ \epsilon_1(r,f)&=-\left(\frac{\gamma}{\delta}\right)\left(\frac{r^2}{r_0^2f^2}\right)pe^{-p},\end{aligned} \quad (21)$$

where we have set $p=\alpha_0E_{00}^2/f^2$. The function p, which is dimensionless and proportional to E_{00}^2, represents the dimensionless beam power on a suitably chosen scale. From Eqs. (16) and (21), the condition for $d^2f/d\xi^2$ to vanish is

$$\rho^2=\rho_0^2f^2=\frac{\delta}{\gamma}\frac{\exp p}{p}. \quad (22)$$

A. Critical curve

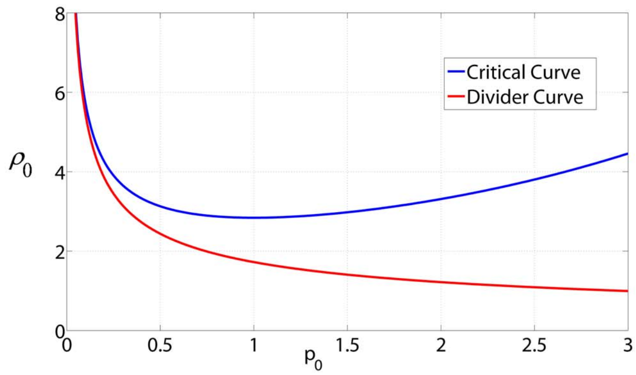
At $\xi=0$ $(f=1)$, Eq. (22) reduces to

$$\rho_0^2=\frac{\delta}{\gamma}\frac{\exp p_0}{p_0}, \quad (23)$$

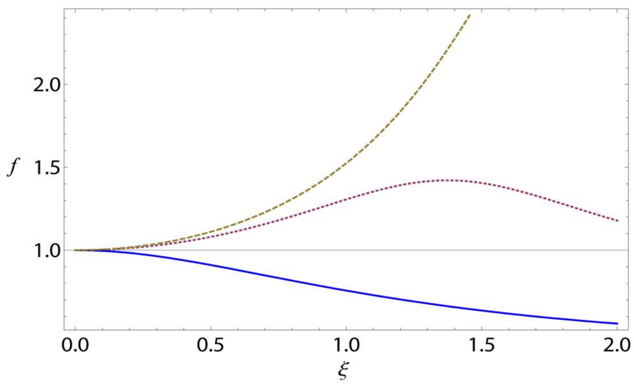
where $p_0=\alpha_0E_{00}^2$ and δ is given by Eq. (5).

If the initial beam power p_0 and initial width ρ_0 satisfy Eq. (23), on account of Eq. (16), the beam will propagate through the plasma without any change in the beam width $(f=1)$. Such a propagation is referred to as the uniform wave guide mode. We shall refer the dependence of ρ_0 versus p_0 according to Eq. (23) as the critical power curve [see, e.g., Fig. 1(a)]. The parameter $(d^2f/d\xi^2)_{\xi=0}$ vanishes if the initial beam power, beam-width point (p_0,ρ_0) falls on the critical curve; it has a positive value if it falls below it and a negative value if the point (p_0,ρ_0) lies above the curve.

Suppose (p_0,ρ_0) lies above the critical curve so that $(d^2f/d\xi^2)_{\xi=0}$ is negative. Therefore, as the beam propagates $df/d\xi$ falls below its initial value zero and f falls below its initial value unity. With decreasing f (below unity) $\rho=\rho_0f$ falls below its initial value ρ_0 and $p=p_0/f^2$ increases above the initial value p_0; hence, the point (p,ρ) of the beam shifts towards a downward right direction as it propagates through the plasma and at some distance must reach the critical curve. Since the point (p,ρ) satisfies Eq. (22), which has exactly the same algebraic form as Eq. (23) the beam-power–beam-width point (p,ρ) on reaching the critical curve ensures vanishing of $(d^2f/d\xi^2)$ at that point. This implies the existence of a point of inflection in the f versus ξ curve; therefore, $df/d\xi$ starts increasing beyond this point (from a



(a)



(b)

FIG. 1. (Color online) (a) Dependence of the dimensionless beam width on the dimensionless beam power for a beam propagating in collisionless magnetized plasma. Critical and divider curves corresponding to $\Omega_p^2=0.2$ and $\Omega_c=0.2$; $p_0=\alpha_0 E_{00}^2$ and $\rho_0=r_0\omega/c$. (b) Dependence of the extraordinary beam-width parameter f on the dimensionless propagation distance $\xi=cz/\delta\omega r_0^2$ in collisionless magnetized plasma, for $p_0=1$ and $\rho_0=0.5$ (------), $p_0=2$ and $\rho_0=1.5$ (······), and $p_0=1$ and $\rho_0=4$ (———).

negative value) and subsequently reaches the value zero, which means that f reaches the minimum value. The beam thus executes oscillatory convergence (self-focusing) such that the beam-width parameter varies between its initial value unity and a minimum value.

In case the initial point (p_0,ρ_0) of the beam falls below the critical curve, as the beam propagates the point (p,ρ) moves in the left upward direction and may reach the critical curve. The condition for the point (p,ρ) to reach the critical curve can be obtained from Eq. (22) as

$$\rho^2 p = \rho_0^2 p_0 = \frac{\delta}{\gamma}\exp\left(\frac{p_0}{f^2}\right), \tag{24}$$

B. Divider curve

For Eq. (24) to yield a positive value of f^2, one must have

$$\rho_0^2 p_0 > \delta/\gamma.$$

Therefore, if the initial beam-power–beam-width point (p_0,ρ_0) lies above the curve represented by the equation

$$\rho_0^2 p_0 = \delta/\gamma \tag{25}$$

the beam-power and beam-width point (p,ρ) will reach the critical curve at some finite value f or at some distance ξ. Obviously the beam with such an initial beam power beam width (p_0,ρ_0) will experience an oscillatory divergence. In case (p_0,ρ_0) of the beam lies on or below this curve [Eq. (25)] the (p,ρ) value of the beam will not reach the critical curve and hence the beam will diverge steadily. The curve represented by Eq. (25) is referred as the divider curve.

We have thus clearly distinct cases covering the entire range of values of p_0, ρ_0 constituting the positive quadrant of the p_0-ρ_0 space, as shown in Fig. 1(a).

(i) (p_0,ρ_0) lying on the critical curve: The beam propagates in the uniform wave guide mode.
(ii) (p_0,ρ_0) lying above the critical curve: The beam undergoes oscillatory convergence with the beam width varying between the initial value $(f=1)$ and a minimum.
(iii) (p_0,ρ_0) lying between the critical and divider curves: The beam undergoes oscillatory divergence with the beam width varying between the initial value $(f=1)$ and a maximum.
(iv) (p_0,ρ_0) lying on or below the divider curve: The beam diverges steadily.

Figure 2(a) illustrates the dependence of the critical and divider curves on the magnitude of static magnetic field (via $\Omega_c=\omega_c/\omega=eB_0/m\omega c$) applied to the plasma. Both critical and divider curves get lower as Ω_c increases because of enhanced nonlinearity.

Equation (15) with $\epsilon_0(z)$ and $\epsilon_1(r,z)$ given by Eq. (21), has been integrated numerically using the initial conditions Eq. (16) for several points in the (p_0,ρ_0) space. The f versus ξ graphs [as shown in Fig. 1(b)] obtained for these points agree with the conclusions given above. The dependence of the beam-width parameter on the dimensionless propagation distance for various values of $\Omega_c=\omega_c/\omega=eB_0/m\omega c$ is shown in Fig. 2(b). The increase of static magnetic field has decreased the diffraction of beam on account of increased nonlinearity and hence the beam gets more focused for stronger magnetic fields.

IV. COLLISIONAL PLASMA

Previous analysis for the collisional magnetized plasma has employed an electron density-temperature relationship derived phenomenologically. Here, we shall rely on the kinetic theoretical result[3] for the number density N as

$$N(T) = N(T_0)\left(\frac{2T_0}{T_0+T}\right)^{1-s/2}.$$

The parameter s in the exponent characterizes the nature of collisions and can be defined through the dependence of collision frequency on the random electron speed v as

Collision frequency $\propto v^s$.

In case of collisions of the electrons with neutral particles, s may be taken as 1, while in the case in which the

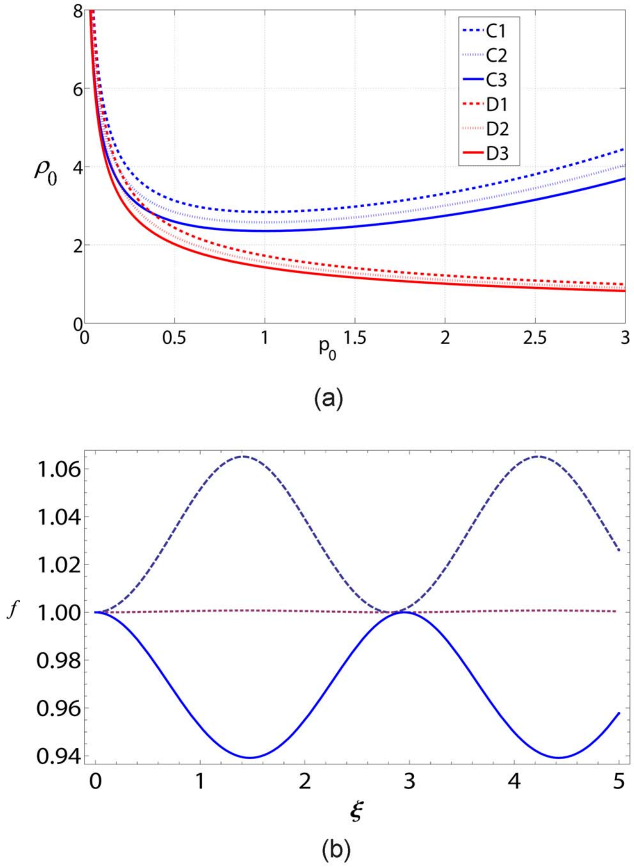


FIG. 2. (Color online) (a) Dependence of the dimensionless beam width on the dimensionless beam power for a beam propagating in collisionless magnetized plasma. Critical curves (upper three, in blue) are depicted, corresponding to $\Omega_p^2=0.2$ and $\Omega_c=0.2$ (C1), $\Omega_c=0.3$ (C2), and $\Omega_c=0.4$ (C3). Divider curves (lower three, in red) are also depicted, corresponding to $\Omega_p^2=0.2$ and $\Omega_c=0.2$ (D1), $\Omega_c=0.3$ (D2), and $\Omega_c=0.4$ (D3), respectively. We have taken $p_0=\alpha_0 E_{00}^2$ and $\rho_0=r_0\omega/c$ everywhere. (b) Dependence of the extraordinary beam-width parameter f on the dimensionless propagation distance $\xi=cz/\delta\omega r_0^2$ in collisionless magnetized plasma for $p_0=1$ and $\rho_0=2.5$ and $\Omega_c=0.2$ (------), $\Omega_c=0.3$ (⋯⋯), and $\Omega_c=0.4$ (———).

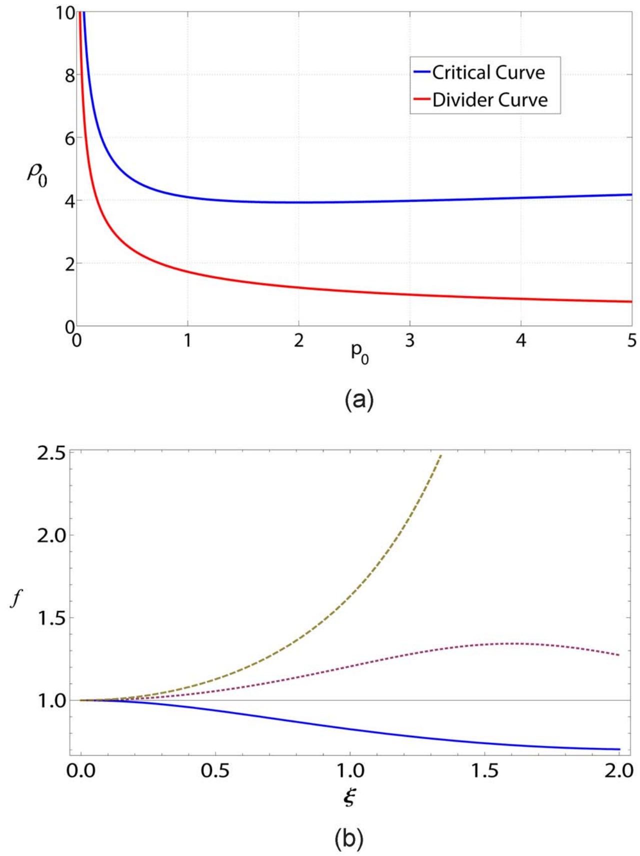


FIG. 3. (Color online) (a) Dependence of the dimensionless beam width on the dimensionless beam power for a beam propagating in collisional magnetized plasma. Critical and divider curves are depicted corresponding to $\Omega_p^2=0.2$ and $\Omega_c=0.2$; $p_0=\alpha_1 E_{00}^2$ and $\rho_0=r_0\omega/c$. (b) Dependence of the extraordinary beam-width parameter f on the dimensionless propagation distance $\xi=cz/\delta\omega r_0^2$ in collisional magnetized plasma for $p_0=0.5$ and $\rho_0=1$ (------), $p_0=1.5$ and $\rho_0=2$ (⋯⋯), and $p_0=1$ and $\rho_0=5$ (———).

collision takes place between electrons and ions, $s=-3$. If the collision frequency does not depend on v, then $s=0$. The temperature is related to the electromagnetic field and is given by

$$\frac{T}{T_0}=1+\frac{\alpha}{2}\frac{E_+E_+^*}{(1-\Omega_c)^2},$$

where α is given by Eq. (20).

Using these two relations, one obtains

$$\frac{N(T)}{N(T_0)}=\left[1+\frac{1}{4}\frac{\alpha E_+E_+^*}{(1-\Omega_c)^2}\right]^{s/2-1}. \tag{26}$$

Remembering that $\Omega_p^2(T)\propto N(T)$, from Eq. (26), one obtains

$$\epsilon_+=1-\gamma\left[1+\frac{1}{4}\frac{\alpha E_+E_+^*}{(1-\Omega_c)^2}\right]^{s/2-1}, \tag{27}$$

where γ is given by Eq. (18).

Using Eqs. (3), (14), and (27), one gets

$$\epsilon_0(z)=\left(\frac{1}{\delta}\right)[1-\gamma(1+p)^{s/2-1}] \tag{28}$$

and

$$\epsilon_1(r,z)=-\left(\frac{\gamma}{\delta}\right)p(1+p)^{s/2-1}\left(1-\frac{s}{2}\right)\left(\frac{r^2}{r_0^2f^2}\right), \tag{29}$$

where δ is given by Eq. (5) and p is expressed as

$$p=\alpha_1E_{00}^2/f^2, \tag{30}$$

where

$$\alpha_1=\alpha/4(1-\Omega_c)^2. \tag{31}$$

In a way similar to that adopted for a collisionless plasma, the critical curve is seen to satisfy the equation

$$\rho_0^2p_0=\left(\frac{2}{(2-s)}\frac{\delta}{\gamma}(1+p_0)^{2-s/2}\right), \tag{32}$$

and the divider curve is represented by

$$\rho_0^2p_0=2\delta/\gamma(2-\mathrm{s}). \tag{33}$$

The critical and divider curves in the positive quadrant of the p_0-ρ_0 space for collisional magnetized plasma are

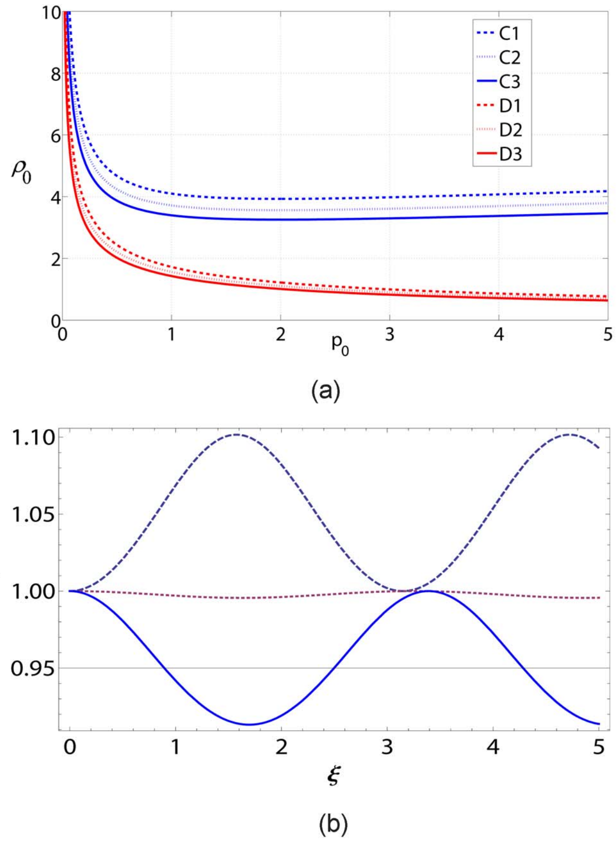


FIG. 4. (Color online) (a) Dependence of the dimensionless beam width on the dimensionless beam power for a beam propagating in collisional magnetized plasma. Critical curves (upper three, in blue) are depicted, corresponding to $\Omega_p^2=0.2$ and $\Omega_c=0.2$ (C1), $\Omega_c=0.3$ (C2), and $\Omega_c=0.4$ (C3). Divider curves (lower three, in red) are also depicted, corresponding to $\Omega_p^2=0.2$ and $\Omega_c=0.2$ (D1), $\Omega_c=0.3$ (D2), and $\Omega_c=0.4$ (D3), respectively. We have taken $p_0=\alpha_0 E_{00}^2$ and $\rho_0=r_0\omega/c$ everywhere. (b) Dependence of the extraordinary beam-width parameter f on the dimensionless propagation distance $\xi=cz/\delta\omega r_0^2$ in collisional magnetized plasma for $p_0=1$ and $\rho_0=3$, and $\Omega_c=0.2$ (------), $\Omega_c=0.3$ (⋯⋯), and $\Omega_c=0.4$ (———).

shown in Fig. 3(a). The dependence of beam-width parameter on the dimensionless propagation distance is shown in Fig. 3(b) corresponding to the (p_0,ρ_0) lying above the critical curve, (p_0,ρ_0) lying between the critical and divider curves, and (p_0,ρ_0) lying on or below the divider curve. Figure 4(a) illustrates the dependence of critical and divider curves on the magnitude of static magnetic field (i.e., $\Omega_c=\omega_c/\omega=eB_0/m\omega c$) applied to the plasma. The critical and divider curves get lower as Ω_c increases. The dependence of beam-width parameter on the dimensionless propagation distance for various values of $\Omega_c=\omega_c/\omega=eB_0/m\omega c$ is shown in Fig. 4(b).

The results of the present analysis are readily applicable to the ordinary (E_x-iE_y) mode of propagation, when Ω_c is replaced by $(-\Omega_c)$.

For self-focusing to be possible ϵ_1 should be negative [cf. Eq. (29)]; this is so only when $\delta>0$; for the extraordinary mode this condition implies

$$\Omega_p^2 < 2(1-\Omega_c)/(2-\Omega_c). \qquad (34)$$

Further, the studies will be meaningful only when ϵ_{+0} is positive:

$$1-\Omega_c-\Omega_p^2>0; \quad \text{i.e., } \Omega_p^2<1-\Omega_c. \qquad (35)$$

It can easily be seen that for $0<\Omega_c<1$ the inequality (35) implies that Eq. (34) is satisfied.

V. CONCLUSIONS

In summary, we have investigated the propagation of a laser beam in magnetized plasma. We have established the new relationship between critical power and the beam width that explains clearly the beam convergence or divergence of an extraordinary electromagnetic beam. To study the behavior of the beam-width parameter with the propagation distance corresponding to the beam-power–beam-width plane, we have carried out a numerical computation of Eq. (15) for the following parameters:

$$N_e=10^{19}\ \text{cm}^{-3}, \quad \omega=10^{15}\ \text{rad/s}, \quad B_0=11\ \text{MG}.$$

Critical and divider curves have been plotted in the dimensionless beam-width–beam-power plane for a wide range of Ω_p^2 and Ω_c. Typical curves, corresponding to collisionless and collisional $(s=1)$ plasmas have been plotted. These are presented here as Figs. 1(a) and 2(a) for $\Omega_p^2=0.2$ and $\Omega_c=0.2$. The curves get lower as Ω_p^2 and Ω_c increase. This is easily understood in terms of increasing nonlinearity. On account of the assumption $\nu\ll\omega$, the region around gyroresonance has not been explored. The dependence of f on ξ for typical points in the three regions has been worked out and illustrated in Figs. 1(b) and 2(b); self-focusing, oscillatory, and steady divergence, characteristic of the three regions is observed. Figures 3(a) and 4(a) have been plotted to explore the effect of static magnetic field (i.e., $\Omega_c=0.2$, 0.3, and 0.4) on critical and divider curves in the dimensionless beam-width–beam-power plane. The critical power shows a decrease with Ω_c; as Ω_c varies from 0.2 to 0.4, as shown in Figs. 3(a) and 4(a) for collisionless and collision plasmas, respectively. The dependence of f on ξ is shown in Figs. 3(b) and 4(b) for varying magnitude of static magnetic field and hence Ω_c. The application of a magnetic field favors the self-focusing of electromagnetic beam in a plasma.

The analysis presented in this paper will be extended to pulse propagation incorporating dispersion effects, radial variation of electron density and modulation instability of an electromagnetic wave in magnetized plasma.

Our results are relevant in laser-plasma interaction experiments, where a laser beam can be used for plasma heating (as in inertial fusion schemes).

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[1]S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Sov. Phys. Usp. **10**, 609 (1968).
[2]M. S. Sodha, A. K. Ghatak, and V. K. Tripathi, *Self Focusing of Laser Beams in Dielectrics, Plasma and Semiconductor* (Tata McGraw Hill, Delhi, 1994).
[3]M. S. Sodha, A. K. Ghatak, and V. K. Tripathi, Prog. Opt. **13**, 169 (1976).

[4] A. K. Ghatak, I. C. Goyal, and M. S. Sodha, Opt. Acta **19**, 693 (1972).
[5] M. S. Sodha and V. K. Tripathi, Phys. Rev. A **16**, 2101 (1977).
[6] E. Esarey, P. Sprangle, J. Krall, and A. Ting, IEEE J. Quantum Electron. **33**, 1879 (1997).
[7] D. Umstadter, Phys. Plasmas **8**, 1774 (2001).
[8] A. Sharma, G. Prakash, M. P. Verma, and M. S. Sodha, Phys. Plasmas **10**, 4079 (2003).
[9] G. Prakash, A. Sharma, M. P. Verma, and M. S. Sodha, J. Opt. Soc. Am. B **22**, 1268 (2005).
[10] M. S. Sodha and A. Sharma, Phys. Plasmas **13**, 053105 (2006).
[11] M. S. Sodha, S. K. Agarwal, and A. Sharma, Phys. Plasmas **13**, 103107 (2006).
[12] P. Jha, R. K. Mishra, G. Raj, and A. K. Upadhyay, Phys. Plasmas **14**, 053107 (2007).
[13] P. Sprangle, E. Esarey, A. Ting, and J. Joyce, Appl. Phys. Lett. **53**, 2146 (1988).
[14] Y. Avitzour and G. Shvets, Phys. Rev. Lett. **100**, 065006 (2008).
[15] A. V. Gurevich, *Nonlinear Phenomena in Ionosphere* (Springer, New York, 1978), pp. 258–281.
[16] M. S. Sodha, A. Sharma, M. P. Verma, and M. Faisal, Phys. Plasmas **14**, 052901 (2007).
[17] S. K. Agarwal and A. Sharma, J. Atmos. Sol.-Terr. Phys. **70**, 980 (2008).
[18] M. S. Sodha, R. S. Mittal, S. Kumar, and V. K. Tripathi, Opto-electronics (London) **6**, 167 (1974).
[19] C. Chen, J. A. Davies, G. Zhang, and J. S. Wurtele, Phys. Rev. Lett. **69**, 73 (1992).
[20] M. S. Sodha, R. K. Khanna, and V. K. Tripathi, J. Phys. D **7**, 2188 (1974).
[21] D. P. Tewari and A. Kumar, Plasma Phys. **17**, 133 (1975).
[22] A. G. Litvak, Sov. Phys. JETP **30**, 344 (1970).
[23] D. P. Tewari, H. D. Pandey, A. K. Agarwal, and V. K. Tripathi, J. Appl. Phys. **44**, 3153 (1973).
[24] M. S. Sodha, S. K. Sinha, and R. P. Sharma, J. Phys. D **12**, 1079 (1979).
[25] M. S. Sodha, J. K. Sharma, S. Kumar, and V. K. Tripathi, J. Appl. Phys. **50**, 2638 (1979).
[26] M. S. Sodha and A. Sharma, J. Plasma Phys. **74**, 473 (2008).
[27] H. A. Salih, V. K. Tripathi, and B. K. Pandey, IEEE Trans. Plasma Sci. **31**, 324 (2003).
[28] V. L. Ginzburg and A. V. Gurevich, Sov. Phys. Usp. **3**, 115 (1960).