Evolution of linearly polarized electromagnetic pulses in laser plasmas

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Abstract

An analytical and numerical investigation is presented of the behavior of a linearly polarized electromagnetic pulse as it propagates through a plasma. Considering a weakly relativistic regime, the system of one dimensional fluid-Maxwell equations is reduced to a generalized nonlinear Schrödinger (GNLS) type equation, which is solved numerically using a split step Fourier method. The spatio-temporal evolution of an electromagnetic pulse is investigated. The evolution of the envelope amplitude of density harmonics is also studied. An electromagnetic pulse propagating through the plasma tends to broaden due to dispersion, while the nonlinear frequency shift is observed to slow down the pulse at a speed lower than the group velocity. Such nonlinear effects are more important for higher density plasmas.The pulse broadening factor is calculated numerically, and is shown to be related to the background plasma density. In particular, the broadening effect appears to be stronger for dense plasmas. The relation to existing results on electromagnetic pulses in laser plasmas is discussed.

I. INTRODUCTION

The propagation of electromagnetic pulses through a plasma is an important area of research due to its relevance to laser driven fusion and laser driven particle accelerators. High power electromagnetic pulse propagation is associated with a wide variety of interesting phenomena, including wake-field generation, relativistic self focusing, harmonic generation, frequency shifting, relativistic soliton generation and collapse[1–9], among others.

Several analytical investigations dealing with the nonlinear interaction of the electromagnetic waves with plasmas have been considered in the past. The interaction of relativistic electromagnetic waves with plasmas was first investigated by Akhiezer and Polovin[10], who pointed out the nonlinear coupling between plasma waves and plane electromagnetic waves. The propagation of circularly polarized electromagnetic pulses was extensively investigated analytically within a one dimensional fully relativistic fluid model and by particle in cell simulations. Exact relativistically intense optical standing waves were considered in Ref. 11 in the framework of a fluid model. The problem of one dimensional circularly polarized relativistic electromagnetic soliton propagation in a cold plasma has been thoroughly investigated by Kozlov *et al.*[12] A class of exact one-dimensional solutions for modulated light pulses coupled to electron plasma waves in a cold plasma was obtained and investigated in Ref. 13. A relativistic electromagnetic soliton solutions with zero group velocity was obtained within a one-dimensional cold plasma model without using the envelope approximation in Ref. 14. The influence of ion motion on soliton structures was investigated in Refs. 15, 16. Stability, mutual collisional interactions, and propagation of one-dimensional circularly polarized electromagnetic solitons in an inhomogeneous plasma were addressed in Ref.17. Interestingly, relativistic effects may lead, via an increase of the electron mass m_e, to a significant enhancement of the refractive index by reducing the local plasma frequency, since the plasma frequency varies as $\omega_{p0} \sim m_e^{-1/2}$. [6,8,10]

The propagation of a linearly polarized transverse wave in a plasma is known to generate longitudinal oscillations when nonlinearities are included.[10] These oscillations are the result of the figure eight motion, caused by the relativistic Lorentz force. The velocity of the electrons, as they undergo figure eight motion, can be expanded into odd harmonics in the transverse direction and even harmonics in the longitudinal direction[5]. Therefore, because of harmonic generation, the interaction of linearly polarized electromagnetic waves with plasmas is more complex than

that of circularly polarized ones. In this case, based on a one-dimensional model, a perturbation expansion is carried out to solve the equations describing a linearly polarized weakly nonlinear laser pulse propagating in the plasma in which the electrons are treated relativistically and ions are assumed stationary. Kuehl *et al*[18] have used a reductive perturbation method, resulting in a hierarchy of equations, the first level of which was shown to lead to a set of two coupled equations for the envelope(s) of the lowest-order vector potential (first harmonic) and the lowest-order scalar potential (zero harmonic). For a circularly polarized electromagnetic beam, only the first harmonic of the vector potential and the zeroth harmonic of the density (scalar potential) exist. It must be stressed that Kuehl *et al*[18] obtained these results under the assumption that the plasma frequency ω_{p0} is much less than the laser frequency ω_0 (i.e. $\omega_{p0}/\omega_0 \ll 1$). Interestingly, Kuehl *et al*[18] also analyzed the very motivation of the perturbative scaling they employed, in terms of the strength of the group dispersion, and thus succeeded in explaining the apparent contradiction among the underdense laser-plasma system outlined above (shown to evolve on a very slow time scale $\epsilon^4 t$) and plasma waves (evolving on $\epsilon^2 t$)[19], wherein reductive perturbation theory leads to a nonlinear Schrödinger type equation instead[20]. In this framework, the authors of Ref.18 also numerically investigated collisions among electromagnetic solitary waves.[21]

The existence and stability of one-dimensional electromagnetic solitons formed in a weakly relativistic linearly polarized laser beam interacting with an underdense cold plasma were discussed within the slowly varying envelope approximation in Refs. 22, 23. Their work advanced one step forward in that they included higher harmonic components. Stationary electromagnetic soliton solutions were shown analytically to be stable, in agreement with the model simulation. As stressed above, the essential difference in the propagation of linearly polarized electromagnetic pulses versus circularly polarized ones is that in the former case there is harmonic content for both longitudinal and transverse components of field and plasma quantities. The harmonic content affects the nonlinear phase and group velocities of linearly polarized electromagnetic waves in plasmas[24] and can be important to both plasma based radiation sources and plasma based accelerators. Elucidating the propagation and dynamics of strongly relativistic localized electromagnetic structures is of outmost importance in plasma based accelerators. Noting that the infinite wavelength limit (only) has essentially been treated in Ref. 22, 23 [note the absence of the fast carrier space variable in (6) therein, thus essentially retaining only the envelope's motion in space], it appears important to anticipate generalizing the results of Refs. 22 ,23 to travelling localize solutions.

In this article, we investigate the dynamics of a linearly polarized localized electromagnetic excitation propagating in a plasma of arbitrary density (avoiding the scaling restrictions in Ref. 18) by considering a second harmonic contribution in the electron density. We shall fully take into account the existence of both fast (carrier) and slow (envelope) time and space dynamics, thus generalizing the results of Refs. 22, 23. Assuming a weakly relativistic cold plasma model and adopting a slowly varying envelope approximation, the system of Maxwell's and fluid plasma equations is reduced to a generalized nonlinear Scrödinger (GNLS) type equation[29], which incorporates nonlocal nonlinear terms (involving space derivatives). This equation describes the coupling between electromagnetic waves and plasma excitations in the weakly relativistic approximation. Using this equation, the existence and evolution of localized traveling electromagnetic pulses (electromagnetic solitarw waves) is investigated numerically.

This paper is organized as follows. In Sec. II we derive the GNLS type evolution equation for the field amplitude from a set of standard model equations. Section III is devoted to the results of numerical simulation of the GNLS type equation derived in the previous section. Finally, we summarize our results in the concluding Section.

II. DYNAMICAL EQUATIONS

We consider the propagation of a long intense laser pulse in a cold collisionless plasma with fixed ions. We use as a starting point the Maxwell equations, coupled to the relativistic fluid equations for the electrons. The one-dimensional approximation in wave propagation is adopted throughout this text, implying that the radiation spot size is large compared to the plasma wavelength. The one dimensional model has been widely adopted in the literature in the context of laser plasma interaction studies. We consider the case where the electromagnetic field propagates along the x axis ($\partial/\partial y = \partial/\partial z = 0$). The nonlinear electromagnetic wave equation, Poisson's equation, electron continuity equation, and electron momentum equation in the Coulomb gauge read respectively as follows:[17]

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{\partial^2 \mathbf{A}}{\partial x^2} = -\frac{n\mathbf{A}}{\gamma}, \tag{1}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n - 1, \tag{2}$$

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)(\gamma u) = \frac{\partial \phi}{\partial x} - \frac{1}{2\gamma}\frac{\partial A^2}{\partial x} \tag{4}$$

$$\gamma = \sqrt{\frac{1 + A^2}{1 - u^2}} \tag{5}$$

where $\mathbf{A}$ is the transverse component of the vector potential, ϕ is the electrostatic potential, n is the electron density, u is the longitudinal component of the electron velocity, and γ is the relativistic factor. The transverse component of the electron velocity is eliminated by an exact integration i.e. $\mathbf{u}_\perp = \mathbf{A}/\gamma$. The one dimensional model with the Coulomb gauge implies that the longitudinal component of the vector potential is zero for localized solutions. In writing the above equations, we have normalized the vector potential by mc^2/e, the density by the ambient plasma density n_0, the electron velocity by the light velocity c; furthermore, the length is normalized by the skin length c/ω_{p0} (where $\omega_{p0}^2 = 4\pi n_{00} e^2/m_e$), and time is scaled by the plasma period (inverse plasma frequency) (ω_{p0}^{-1}). In a weakly relativistic limit (where $A \ll 1$), the above equations are reduced to a coupled system of equations for the vector potential and for a small electron density perturbation[5,25,26]

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)A(x,t) = -\left(1 + \delta n - \frac{A^2}{2}\right)A(x,t), \tag{6}$$

$$\frac{\partial^2 \delta n}{\partial t^2} + \delta n = \frac{1}{2}\frac{\partial^2}{\partial x^2} A^2 \tag{7}$$

where δn is a small perturbation in electron density. In the right hand side of Eq. (6), the second term corresponds to the ponderomotive force while the third term is the contribution of relativistic effects.

We are looking for linearly polarized solutions of Eqs. (6) and (7). In distinction from the case of circular polarization, linearly polarized waves have even harmonics for density and odd harmonics for vector potential[4,5]. Also as shown in Ref. 18, the vector potential and electron density (scalar potential) possess no harmonics through some order, while the third harmonic component of vector potential appears in one order higher than the second harmonic component of electron density. Therefore in a weakly relativistic case, it is adequate to preserve the first harmonic component of the vector potential and the zero and second harmonics component of the density

$$A(x,t) = \frac{1}{2} a(x,t) e^{i(k_0 x - \omega_0 t)} + c.c., \tag{8}$$

$$\delta n(x,t) = n_0(x,t) + \frac{1}{2} n_2(x,t) e^{2i(k_0 x - \omega_0 t)} + c.c. \tag{9}$$

where $a(x,t)$, and $n_2(x,t)$, are complex envelopes and $n_0(x,t)$ is a real quantity In these expressions ω_0 and k_0 are the frequency and wavenumber of the pulse which are normalized bye ω_{p0} and $k_{p0} = \omega_{p0}/c$ respectively. The validity of the slowly varying envelope hypothesis implies the bandwidth is only a small fraction of the carrier frequency ($\triangle\omega/\omega_0 \ll 1$). This implies that envelopes are slowly varying in time. This approximation is well satisfied in the case of long pulses[28]. Substituting (8) and (9) into (7) and collecting the zero and second harmonic terms we obtain

$$n_0(x,t) = \frac{1}{4}\frac{\partial^2 |a|^2}{\partial x^2} \tag{10}$$

$$n_2(x,t) = \frac{1}{4(1-4\omega_0^2)}\left[\frac{\partial^2(a^2)}{\partial x^2} + 4ik_0\frac{\partial(a^2)}{\partial x} - 4k_0^2a^2\right] \tag{11}$$

where slow time derivatives have been neglected. Inserting (10) and (11) into the wave equation (6) and collecting the first harmonic terms, we obtain the wave equation for the vector potential envelope a,

$$\frac{\partial^2 a}{\partial x^2} + 2ik_0\frac{\partial a}{\partial x} + 2i\omega_0\frac{\partial a}{\partial t} = \frac{(1+8\omega_0^2)|a|^2 a}{8(1-4\omega_0^2)} + \frac{1}{4}\frac{\partial^2|a|^2}{\partial x^2}a + \frac{1}{8(1-4\omega_0^2)}\left(\frac{\partial^2 a^2}{\partial x^2} + 4ik_0\frac{\partial a^2}{\partial x}\right)a^* \tag{12}$$

where a^* denotes the complex conjugate of a. In obtaining the latter equations, we have adopted the slowly time-varying envelope approximation, thus the second time derivative was neglected. We have also used the well-known dispersion relation for electromagnetic waves in plasmas in its normalized form, $\omega_0^2 = 1 + k_0^2$.

At this stage, it is appropriate to compare our results to existing results on standing solitons[22,23,26] , by considering the vanishing wavenumber limit $k_0 \to 0$ (hence $\omega_0 \to 1$). First of all, note that a small quintic nonlinearity contribution was omitted in our Eq. (6) [cf, e.g., Eq. (19) in Ref. 26], since of negligible magnitude for small A. Advancing in our calculation above, note that Eq. (11) recovers exactly (23b) in Ref. 26, while our field amplitude evolution equation (12) recovers exactly (4) in Ref. 23. In our case, however, the difference from the standing wave limit is not only quantitative but also structural, as an extra nonlocal dispersive term appears in our Eq. (12) [to be compared with, say, (4) in Ref. 23]. It is convenient to replace the laboratory frame independent variables (x,t) by the independent variables (ξ,τ), where $\xi = x - vt$ and $\tau = t$. Here $v = \omega_0'(k_0) = k_0/\omega_0$ is the (normalized) group velocity of the electromagnetic wave in the linear regime. The variable ξ represents a spatial coordinate in a frame moving at the (linear) group velocity. Using this transformation, Eq. (12) becomes

$$i\frac{\partial a}{\partial \tau} = -R_1 a_{\xi\xi} + R_2(|a|^2)_{\xi\xi}a + R_3|a|^2 a + R_4(a^2)_{\xi\xi}a^* + iR_5(a^2)_\xi a^* \tag{13}$$

where the subscript ξ denotes differentiation with respect to the moving space coordinate ξ and

$$R_1 = 1/2\omega_0 \tag{14}$$

$$R_2 = 1/8\omega_0 \tag{15}$$

$$R_3 = (1 + 8\omega_0^2)/16\omega_0(1 - 4\omega_0^2) \tag{16}$$

$$R_4 = 1/16\omega_0(1 - 4\omega_0^2) \tag{17}$$

$$R_5 = k_0/4\omega_0(1 - 4\omega_0^2) \tag{18}$$

It can be shown that Eq. (13) conserves the "pulse (soliton) mass"

$$I_0 = \int_{-\infty}^{\infty} |a|^2 \, d\xi \, . \tag{19}$$

The partial-derivative equation (PDE) (13) has the form of a generalized nonlinear Schrödinger (GNLS) equation. Note that, in contrast with the "ordinary" nonlinear Schrödinger (NLS) equation[30], which is characterized by a cubic nonlinearity (only), the GNLS equation also possesses three nonlocal nonlinear terms involving space derivatives. In general, however, the GNLS equation does not belong to the class of integrable equations[31]. Therefore we shall proceed by solving the Eq. (13) numerically.

III. NUMERICAL SIMULATION

A. Numerical method

We have integrated Eq. (13) numerically, by using a second order split step Fourier method[29]. A brief outline of the method follows, while lengthy details – found in Ref. 29 – are omitted here.

Eq. (13) can be formally cast in the general form

$$\frac{\partial a}{\partial \tau} = (L + N)a, \tag{20}$$

where L and N are linear and nonlinear operators, respectively, given by

$$L = iR_1 \frac{\partial^2}{\partial \xi^2}$$

and

$$N = -iR_2(|a|^2)_{\xi\xi} - iR_3|a|^2 - iR_4(2|a|^2\partial_{\xi\xi} + 2a^* a_\xi \partial_\xi) + R_5(2|a|^2\partial_\xi)\,.$$

The main idea in the split step method is to approximate the exact solution of Eq. (20) by solving the purely linear and purely nonlinear equations in a given sequential order, in which the solution of one subproblem is employed as an initial condition for the next subproblem. In the second order version of the method, the solution operator is approximated by

$$a(\xi, \tau + h) = \exp\left(\frac{1}{2}hL\right) \exp\left(hN\right) \exp\left(\frac{1}{2}hL\right) a(\xi, \tau)$$

where h is the (small) time step. To this end, a Fourier method is employed for the spatial discretization of both linear and nonlinear subproblems, and a fourth-order Runge-Kutta scheme is used for the time integration of the nonlinear subproblem.

To check the accuracy of the numerical simulation the quantity I_0 is calculated during the simulation. We note that this quantity was conserved, as anticipated, during the whole numerical simulation up to four decimal digits.

B. Pulse evolution

When an electromagnetic wavepacket propagates in a plasma, its spatial shape and other characteristics such as frequency, wavenumber and Fourier spectrum may vary, and this variation strongly depends to plasma parameters. In our study, we have considered a range of values for the plasma parameters in Table I. In order to investigate these phenomena, we have considered an initial condition consisting of a Gaussian pulse with $\omega = 2 \times 10^{16} Hz$, and $k = 6.666 \times 10^7 m^{-1}$ ($\lambda = 94.248 nm$), corresponding to intensity $I = 1.54 \times 10^{17}$ W cm^{-2} (this values for the pulses do not vary throughout the text), launched into a rarefied plasma whose ambient density corresponds to Set 1 in Table I. The evolution of the pulse with amplitude $a = 0.1$ and width $L = 10$ is shown

in Figs. 1(a) and 2(a). We note that the electromagnetic pulse exhibits some broadening in width, while the peak is slightly shifted to the left; the pulse nevertheless preserves its initial symmetry as it propagates through the plasma. The initial broadening of the pulse due to dispersion [note the first term in the right hand side of Eq. (13)] eventually saturates, as dispersion is balanced by nonlinearity. Pulse symmetry preservation is due to the absence of higher order dispersion. Since we have depicted evolution in a frame moving at the linear group velocity, Fig. 1(a) indicates that the peak travels at a velocity which is slightly slower than the linear group velocity, in fact due to the frequency shift in the electromagnetic pulse. These phenomena are intuitively expected and, in fact, may be thought of as resulting from the variation in the plasma density due to ponderomotive effects [32].

The interaction of the transverse electron velocity with the magnetic field of the electromagnetic pulse causes a longitudinal force; in lowest order, this leads to a density perturbation, which consists of zeroth and second harmonics. The harmonics of the electron density generate from the time evolution of the vector potential of Fig. 1(a), are shown in Fig. 3(a). This figure shows that as time passes, the amplitudes of both the zeroth electron density harmonic n_0 and the second electron density harmonic n_2 become smaller; the zeroth harmonic component eventually fades out while the second harmonic component is broadened and reveals a multi peak structure (internal oscillation).

Figure 4(a) shows the time evolution of the electron density as a wavepacket. As apparent from this figure, the initial single well density profile evolves into a multi-pulse density envelope profile. Contrary to the circularly polarized case, here, due to harmonic generation, there is no correspondence between the density wells and the vector potential humps. The evolution of the vector potential and harmonics of electron density and electron density (as a wavepacket) for a Gaussian pulse with amplitude $a = 0.1$ and pulse width $L = \sqrt{10}$ launched into a plasma with higher density corresponds to Set 2 of Table I are shown in Figs. 1(b) - 4(b). A comparison among Figs. 1(a) and 1(b) shows that as the plasma density increases, the broadening of the electromagnetic pulse becomes faster. In the following, this issue was addressed in more details.

The pulse width is more accurately described by the root-mean-square (RMS) width σ defined as[33]

$$\sigma = [< \xi^2 > - < \xi >^2]^{1/2} \tag{21}$$

where the angle brackets denote averaging over the intensity profile as

$$< \xi^n >= \frac{\int_{-\infty}^{\infty} \xi^n |A(\xi,\tau)|^2 \, d\xi}{\int_{-\infty}^{\infty} |A(\xi,\tau)|^2 \, d\xi} \tag{22}$$

and pulse broadening factor K can be defined as the final to initial width ratio,

$$K = \frac{\sigma}{\sigma_0}. \tag{23}$$

In general, obtaining an analytical solution for the pulse width is impractical unless an analytical solution can be found for the pulse evolution equation. We have presented our numerical results in Fig. 5 for the broadening factor K for a Gaussian pulse with initial width $\sigma_0 = 1.58$ that propagates in the plasma for different densities. It is obvious from this figure that by increasing the background plasma density, rate of broadening increases and saturation time decreases. It should be noted that the evolution of K is expected to depend on the initial pulse profile, and might therefore differ among a Gaussian pulse and, e.g., a sech pulse.

The spatial modulation of the second harmonic in the density distribution as can be seen in Fig. 3(b) is an interesting result. The spatial modulation originates from the existence of the non-local dispersive terms in the right hand side of Eq. 11 for the second density harmonic [note that these vanish for $k_0 = 0$, thus recovering e.g. the simpler expressions in Ref. 23]. For short pulses, this terms are more important than long pulses and this can be deduced from a comparison of Figs. 3(a) and 3(b), (the latter corresponds to a shorter pulse). For this reason modulation in Fig. 3(b) is stronger than in Fig. 3(a).

In summary, the numerical solution of Eq. 13 indicates that the weakly relativistic linearly polarized electromagnetic pulse spreads as moves through the plasma, but preserves its symmetry in the shape, and traveling with a velocity slightly less than the linear group velocity. This kind of localized electromagnetic solutions that travel with a velocity close to the speed of light are relevant to particle acceleration.[34] The stability of such saturated pulses is a very important issue and has been discussed in more detail in Refs. 35. As described in the latter reference, stable solutions open up their potential use as an energy carrier, which is an issue of crucial relevance to the fast ignition scheme of laser fusion.

IV. SUMMARY

To summarize, we have considered the interaction of weakly relativistic electromagnetic pulses with a plasma by numerical solving the GNLS equation obtained from Maxwell and cold electron fluid equations. We have investigated the existence and evolution of electromagnetic pulse with group velocity nearly equal to the velocity of light in different regimes of the plasma density. We have found that as the pulse propagates through the plasma it is broadened and travels at a velocity less than the linear group velocity. The spreading of the pulse is related to the second order dispersion term in Eq. 13 and frequency downshift causes the pulse to travel at a velocity less than linear group velocity. It is found that the broadening of the pulse is faster in higher density plasmas.

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TABLE I: Plasma parameters

	Set 1	Set 2
$n_{e00}(cm^{-3})$	1.965×10^{21}	1.397×10^{22}
$\omega_{p0}(Hz)$	2.50×10^{15}	6.667×10^{15}
ω_0(normalized)	8.0	3.0
k_0(normalized)	7.937	2.828
R_1	6.250×10^{-2}	1.666×10^{-1}
R_2	1.562×10^{-2}	4.167×10^{-2}
R_3	-1.571×10^{-2}	-4.346×10^{-2}
R_4	-3.063×10^{-5}	-5.952×10^{-4}
R_5	9.688×10^{-4}	6.546×10^{-3}

Figure Captions

- FIG. 1. Snapshots of the time evolution of the vector potential envelope for a incident laser pulse with $a = 0.1$ (in dimensionless unit) corresponding to $I = 1.54 \times 10^{17} W cm^{-2}$. Plasma parameters as in Table I: (a) Set 1; (b) Set 2.(Color online.)

- FIG. 2. Spatio-temporal evolution of the vector potential envelope for the same conditions as in Fig. 1. Plasma parameters as in Table I: (a) Set 1; (b) Set 2.(Color online.)

- FIG. 3. Time evolution of the envelopes of electron density harmonics n_0 and n_2 for the same conditions as in Fig. 1. Plasma parameters as in Table I: (a) Set 1; (b) Set 2.(Color online.)

- FIG. 4. Snapshots of the time evolution of the electron density profile for the same conditions as in Fig. 1. Plasma parameters as in Table I: (a) Set 1; (b) Set 2.(Color online.)

- FIG. 5. Variation of the pulse-broadening factor K versus propagation time in the plasma for deferent densities: (a) $\omega_{pe} = 0.25 \times 10^{16} Hz$; (b) $\omega_{pe} = 0.33 \times 10^{16} Hz$; (c) $\omega_{pe} = 0.5 \times 10^{16} Hz$; (d) $\omega_{pe} = 1.0 \times 10^{16} Hz$. (Color online.)

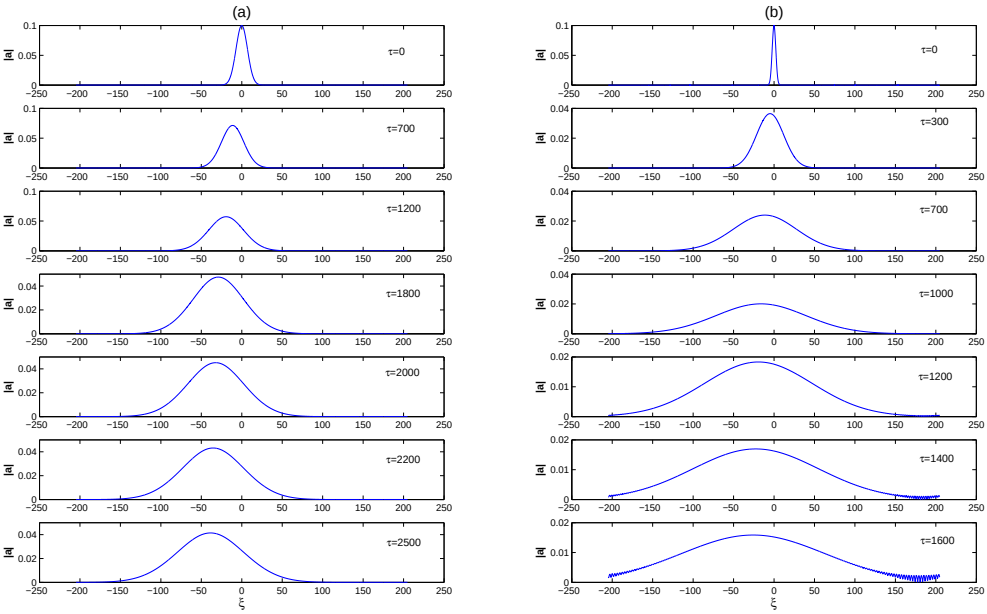
(a)
(b)
|a|
ξ
τ=0
τ=700
τ=1200
τ=1800
τ=2000
τ=2200
τ=2500
τ=0
τ=300
τ=700
τ=1000
τ=1200
τ=1400
τ=1600

FIG. 1:

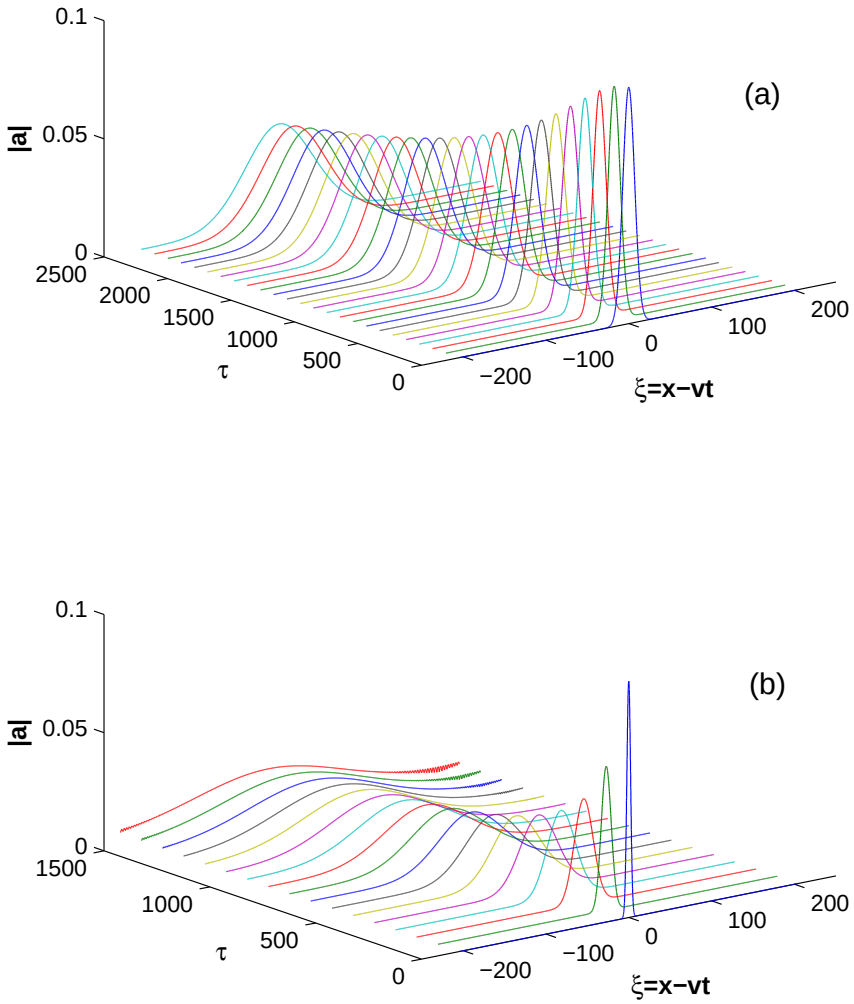
(a)
|a|
0.1
0.05
0
2500
2000
1500
1000
500
0
τ
−200
−100
0
100
200
ξ=x−vt
(b)
|a|
0.1
0.05
0
1500
1000
500
0
τ
−200
−100
0
100
200
ξ=x−vt

FIG. 2:

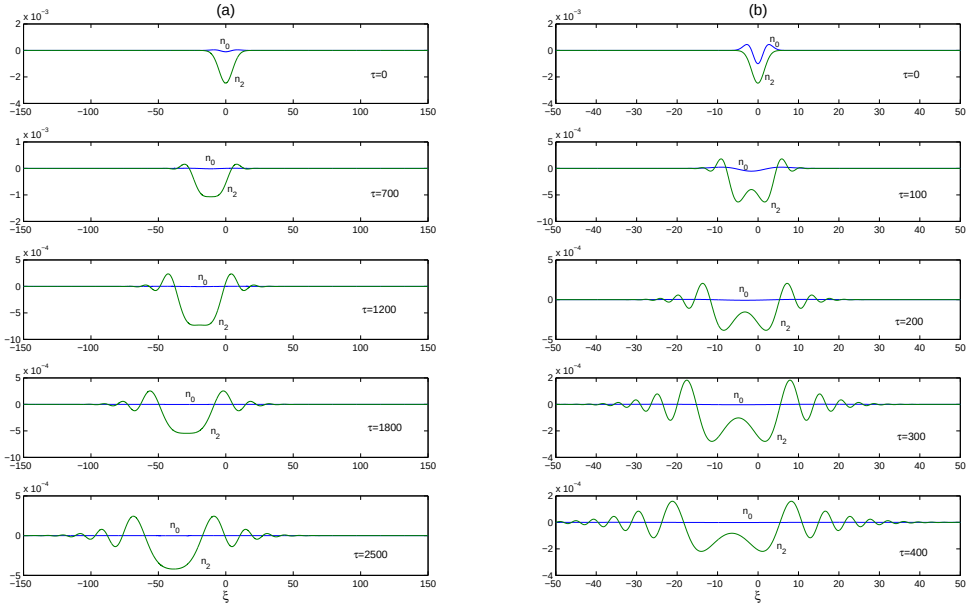

FIG. 3:

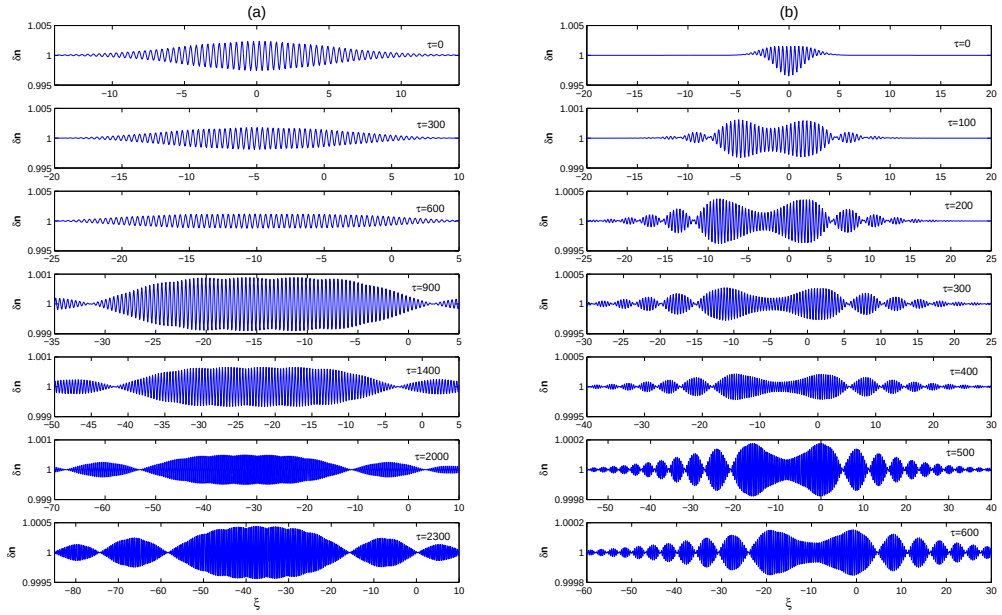
(a)
(b)
δn
ξ
τ=0
τ=300
τ=600
τ=900
τ=1400
τ=2000
τ=2300
τ=0
τ=100
τ=200
τ=300
τ=400
τ=500
τ=600

FIG. 4:

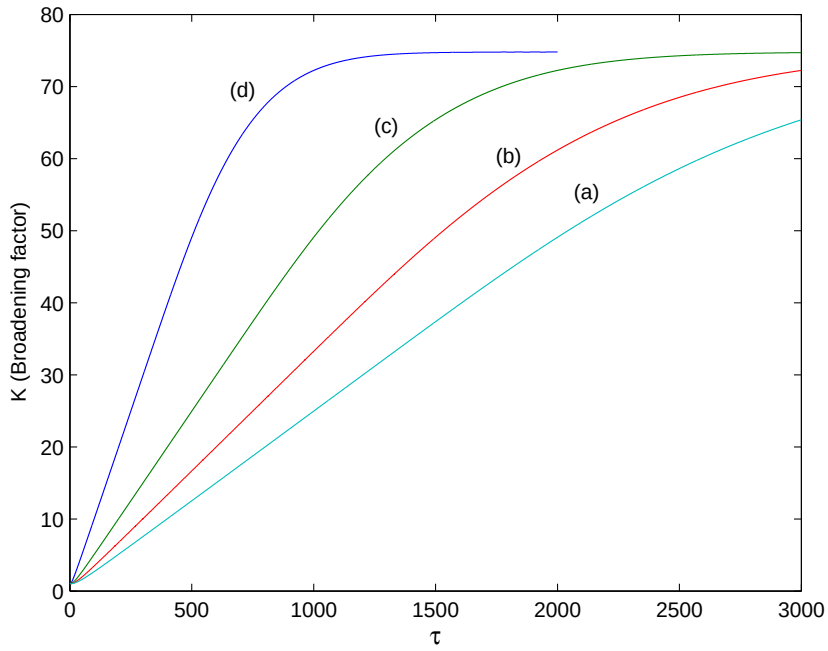
80
70
60
50
40
30
20
10
0
0
500
1000
1500
2000
2500
3000
τ
K (Broadening factor)
(d)
(c)
(b)
(a)

FIG. 5: