

# Stability of dust lattice modes in the presence of charged dust grain polarization in plasmas <sup>\*</sup>

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## Abstract

We discuss the effect of the attractive force associated with overlapping Debye spheres on the dispersion properties of the longitudinal and transverse dust lattice waves in strongly coupled dust crystals. The dust grain attraction is shown to contribute to a destabilization of the longitudinal dust lattice oscillations. The (optic-like) transverse mode dispersion law is shown to change, due to the Debye sphere dressing effect, from the known inverse-dispersive (“backward wave”) form into a normal dispersive law (i.e. the group velocity changes sign). The stability of one-dimensional bi-layers, consisting of (alternating) negatively and positively charged dust particles, is also discussed. The range of parameter values (mainly in terms of the lattice parameter  $\kappa$ ) where the above predictions are valid, are presented.

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## I. INTRODUCTION

An increasing amount of research has recently been devoted in the study of large ensembles of interacting charged particles (plasmas) contaminated by the presence of charged mesoscopic dust particulates [1]. The properties of such plasmas, which can be found in abundance in space, but also in the earth's atmosphere, in fusion devices and in purpose-designed laboratory experiments (see in Ref. [1] for a review) are strongly modified by the presence of charged dust grains, and novel physical mechanisms have so far been shown to come into play.

Of particular importance is the possibility for the occurrence of strongly-coupled crystalline-like dust configurations, due to the strong inter-grain interactions (in combination with the low inertia of the massive charged dust grains) [2]. These dust quasi-lattices are now known to support a variety of linear modes, in addition to nonlinear excitations, which may be of potential use in future applications. Quite naturally, the characteristics of these dust lattice excitations, and also the dust crystal stability itself *par excellence*, depend strongly on the nature of the interactions among the charged mesoscopic dust grains.

At a first approach, *ab initio* studies show that inter-grain electrostatic interactions may be considered to be of the screened Coulomb (Debye - Hückel) type [3]. More refined theoretical studies have later shown that taking into account plasma polarization due to the sheath region (near the grain surface) associated with the grains [4, 5] results in a strong modification of the (oppositely charged) charge cloud surrounding the particles. This “dressing” effect leads to a change in the very nature of the inter-particle interactions, which may even become attractive for equal-sign charged particles (inversely, repulsive interactions may appear in the case of opposite neighboring grain charges). It may be mentioned, for completeness, that a similar effect is observed if one takes into account the influence of the supersonic ion flow (towards the negative electrode, in the sheath region accommodating the dust crystal, during gas discharge experiments) on the interactions (“ion wake” effect), in the vicinity of the electrode (acting as a conducting wall) (see e.g. in Ref. [6]); nevertheless, ion wake effects will be neglected here.

The electrostatic dressing effect was recently claimed to bear a destabilizing effect on transverse dust-lattice collective oscillations (dust-lattice waves, DLWs) [7], in a specific range of values for the lattice (inter-grain) spacing, as compared to the effective dusty

plasma Debye radius.

The purpose of this Short Communication is two-fold. First, we aim to investigate the influence of the dressing effect on the transverse and longitudinal dust-lattice oscillations; we shall show that *longitudinal* dust-lattice waves (LDLWs) may be destabilized by this effect, while the *transverse* dust-lattice wave (TDLW) dispersion law may suffer a structural change from inverse (backward) to normal optic-like dispersion. No TDLW instability is predicted, contrary to the (erroneous) results of Ref. [7]. Second, we shall point out that the inverse effect, namely oscillation *stabilization* due to dressed electrostatic interactions, may be anticipated in (Wigner) crystals consisting of opposite-charge-sign charged dust particles.

## II. DRESSED ELECTROSTATIC INTERACTIONS: PREREQUISITES

The established form of the potential (energy) of interaction between two particles (charges  $Q_1$  and  $Q_2$ ) located at a distance  $r$  reads [1, 4, 5, 7]

$$U_{drD}(r) = Q_1 Q_2 \frac{e^{-r/\lambda_D}}{r} \left( 1 - \delta \frac{r}{2\lambda_D} \right) \equiv \left( \frac{|Q_1 Q_2|}{\lambda_D} \right) s \frac{e^{-x}}{x} \left( 1 - \delta \frac{x}{2} \right), \quad (1)$$

where  $x = r/\lambda_D \equiv \kappa r'$ , and  $\lambda_D$  denotes the effective Debye radius [1, 3]; here, we have defined the lattice parameter  $\kappa = r_0/\lambda_D$  and the reduced space variable  $r' = r/r_0$ . The parameter  $s = \text{sgn}(Q_1 Q_2) = \pm 1$  is equal to 1 (-1) for equal- (opposite-)charge-sign particles, respectively. The parameter  $\delta$  simply takes the values 1 (for “dressed” Debye interactions) and 0 (recovering the familiar unperturbed Debye form); unless otherwise stated, we shall keep  $\delta = 1$  in the following.

The potential form (1) (for  $\delta = 1$ ), studied in Refs. [1, 4, 5, 7], is depicted in Fig. 1. For  $s = 1$  (equal charge-sign grains), it changes sign at  $x = 2$ , shifting from repulsive to attractive interactions (among equal charge signs, here). Furthermore, it bears a minimum at  $x = 1 + \sqrt{3} \approx 2.732$ , which may play the role of a potential well for neighboring particles located at an appropriate distance; naturally, this potential form was suggested as a simple model for dust molecule formation in earlier works [1, 5].

### III. TRANSVERSE DUST-LATTICE WAVES

The dispersion relation for transverse dust-lattice oscillations is given by [1, 8]

$$\omega_T^2 = \omega_g^2 - 4\omega_{T,0}^2 \sin^2(kr_0/2), \quad (2)$$

where  $r_0$  is the lattice spacing and  $k$  is the wavenumber. The gap frequency  $\omega_g = \lim_{k \rightarrow 0} \omega_T(k)$  is related to the plasma sheath environment (assumed to form a parabolic potential in the transverse direction, centered at the crystal levitation height), and need not be discussed here; we retain the condition  $\omega_g^2/\omega_{T,0}^2 > 4$ , which should be imposed for stability (so that  $\omega_T^2 > 0$  in the entire Brillouin zone  $[0, \pi/r_0]$ ). The characteristic constant  $\omega_{T,0}^2$  (which bears dimensions of a squared frequency) is related to the interaction potential as

$$\omega_{0,T}^2 = -U'(r_0)/(Mr_0), \quad (3)$$

where  $M$  denotes the dust grain mass. Combining with Eq. (1), one obtains

$$\omega_{0,T(drD)}^2 = s \left( \frac{|Q_1 Q_2|}{M\lambda_D^3} \right) \frac{e^{-\kappa}}{\kappa^3} \left( 1 + \kappa - \delta \frac{\kappa^2}{2} \right). \quad (4)$$

The right-hand side changes sign at the potential extremum, viz.  $U'(r_0) = 0$ . Specifically, for  $s = 1$  (equal-sign charges),  $\omega_{0,T(drD)}^2$  will be a positive (negative) quantity for values of  $\kappa$  below (above)  $\kappa_1 = 1 + \sqrt{3} \approx 2.732$  (and the inverse qualitative picture holds for  $s = -1$ ). This behaviour is depicted in Fig. ???. We note that this change in sign is not possible for  $\delta = 0$  (simple Debye case), where the well-known (positive) form  $\omega_{0,T}^2 = [|Q_1 Q_2|/(M\lambda_D^3)] e^{-\kappa} (1 + \kappa)/\kappa^3$  is recovered [1, 8].

The change in the sign of  $\omega_{0,T(drD)}^2$  at  $\kappa = \kappa_1$  was (correctly) pointed out in Ref. [7], where our Eq. (4) was elaborated – cf. Eq. (4) therein – yet was misinterpreted as a source of instability (see that an acoustic behaviour was erroneously attributed to TDLWs therein). Upon inspection of Eq. (2), we see that what actually happens is a structural change in the dispersion curve, which obtains a normal (inverse) optic-like form for negative (positive) values of  $\omega_{0,T(drD)}^2 \sim -U'(r_0)$ . This behaviour is depicted in Fig. ???. We conclude that transverse dust-lattice waves may lose their long-discussed (and experimentally confirmed) *backward-wave* property (viz. group and phase velocities of opposite signs) if the lattice parameter  $\kappa$  attains values higher than  $\kappa_1$ ; this situation has not yet been investigated experimentally.

#### IV. LONGITUDINAL DUST-LATTICE WAVES

The dispersion relation for longitudinal dust-lattice oscillations is given by

$$\omega_L^2 = 4\omega_{L,0}^2 \sin^2(kr_0/2), \quad (5)$$

where  $r_0$  and  $k$  have been defined above. Contrary to TDLWs, here  $\omega_L(k)$  goes to zero as  $\omega_L(k) \approx \omega_{L,0}r_0k \equiv c_s k$ , where  $c_s$  is the LDL sound speed. The characteristic constant  $\omega_{T,0}^2$  (which bears dimensions of a squared frequency) is related to the interaction potential as [1, 8]

$$\omega_{0,L}^2 = U''(r_0)/M. \quad (6)$$

Combining with Eq. (1), one obtains

$$\omega_{0,T(drD)}^2 = 2s \left( \frac{|Q_1 Q_2|}{M\lambda_D^3} \right) \frac{e^{-\kappa}}{\kappa^3} \left( 1 + \kappa + \frac{\kappa^2}{2} - \delta \frac{\kappa^3}{4} \right). \quad (7)$$

The right-hand side changes sign at the potential deflection point, viz.  $U''(r_0) = 0$ . In specific, for  $s = 1$  (equal-sign charges),  $\omega_{0,L(drD)}^2$  will be a positive (negative) quantity for values of  $\kappa$  below (above)  $\kappa_2 \approx 3.48$  (and the inverse qualitative picture holds for  $s = -1$ ). This behaviour is depicted in Fig. ???. Similar to the TDLW case above, we note that this change in sign is not possible for  $\delta = 0$  (simple Debye case), where the known (positive) expression  $\omega_{0,L}^2 = 2[|Q_1 Q_2|/(M\lambda_D^3)] e^{-\kappa} (1 + \kappa + \kappa^2/2)/\kappa^3$  is recovered [1, 8].

We draw the conclusion that for  $\kappa$  values above  $\kappa_2$ , i.e. resulting in negative values of  $\omega_{0,L(drD)}^2 \sim U''(r_0)$ , longitudinal dust-lattice oscillations will be destabilized. For lower  $\kappa$  values, LDLWs will be stable; such is reported to be the case in present day experiments, where  $\kappa \approx 1$  or slightly higher.

#### V. STABILIZATION OF LDL WAVES IN CRYSTALS OF ALTERNATING CHARGE-SIGN GRAINS

An interesting consequence of the electrostatic potential “dressing” effect is the following. Let us consider a one-dimensional lattice of charged dust grains of alternating charge sign, i.e. following the pattern:

$$\dots, +, -, +, -, +, -, +, -, +, \dots$$

Naturally, Coulomb-like interactions are expected to be attractive. Interestingly, this fact would in principle give rise to unstable longitudinal displacements: indeed, as one may

easily check, any small horizontal displacement from the equilibrium position would result in lattice disruption (melting), since the resultant total force (mutual attraction from both neighbours) would point outwards, and would thus not act as a restoring force.

Taking into account the dressed Debye potential given by Eq. (1), for opposite grain charge-signs, i.e. for  $s = -1$ , one essentially obtains an reversed, qualitatively speaking, picture, as compared with the case  $s = +1$  treated above; cf. Fig. 1, upon setting  $U \rightarrow -U$ , which yields the mirror-symmetric plot, with respect to the horizontal axis; the corresponding figure is omitted here, for brevity. Most interestingly, considering this type of interaction among one (any) grain and its first order neighbours, we see that the total force  $F_n = F_{n-1,n} + F_{n+1,n}$  felt by the  $n$ -th grain, viz.

$$F_n = -\left[\frac{\partial U_{n-1,n}}{\partial z_n} + \frac{\partial U_{n+1,n}}{\partial z_n}\right] = -\frac{\partial}{\partial z_n}[U_{drD}(r_0 + z_n) + U_{drD}(r_0 - z_n)] \equiv -\left[\frac{\partial}{\partial x}U_{total}(x)\right]_{x=z_n}$$

derives from a total potential, say  $U_{total}(x)$ , which may here, for  $\delta = 1$ , present a local minimum (hence a stable equilibrium position for the  $n$ -th grain). Indeed, upon a simple numerical investigation, one finds that the local extremum at  $x = 0$ , viz.  $U'_{total}(0) = 0$ , is a local minimum (maximum), i.e.  $U''_{total}(0)$  is positive (negative) for  $\kappa$  values above (below) a critical value  $\kappa_3 \approx 3.4798$ . This qualitative behavior is depicted in Fig. ???. Therefore, the electrostatic dressing effect may result in stabilization of longitudinal grain displacements in a bi-lattice, consisting of oppositely charged neighboring grains. Nevertheless, this is true for a lattice spacing above  $\approx 3.4798$  times the Debye length  $\lambda_D$ . Remarkably, this possibility is not provided in the absence of the dressing effect. Indeed, analyzing the form of  $U_{total}(x)$  in the case  $\delta = 0$  (i.e. for simple, unperturbed Debye interactions), one sees that no stable equilibrium point occurs in this case; cf. Fig. ???.

## VI. CONCLUSIONS

We have analyzed, from first principles, the stability of transverse and longitudinal dust-lattice oscillations, by taking into account the electrostatic (quasi-Debye) dressing effect, i.e. the polarization due to the sheath surrounding the surface of charged grains. Our results are summarized as follows.

First, transverse dust-lattice waves are characterized by an inverse (normal) dispersive frequency-wavenumber dependence, for values of the lattice parameter  $\kappa$  below (above) a

threshold  $\kappa_1$  (see above); no instability is observed in this case (contrary to the arguments in Ref. [7]).

Second, longitudinal dust-lattice waves are intrinsically stable (unstable) for values of  $\kappa$  below (above) a critical value  $\kappa_2$  (see above); crystal disruption (melting) is expected beyond this threshold.

Finally, the dressing effect may stabilize the (otherwise inherently unstable) longitudinal oscillations in a bi-crystal, consisting of dust grains of alternating charge-signs.

It may be noted that most of the reported dust-crystal experiments have focused in the neighbourhood of  $\kappa \approx 1$  (or slightly above this value). In principle, however, by adjusting tunable plasma parameters, our predictions herein may be experimentally checked and, hopefully, confirmed.

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Support from the *Deutsche Forschungsgemeinschaft* through *Sonderforschungsbereich (SFB) 591 – Universelles Verhalten Gleichgewichtsferner Plasmen: Heizung, Transport und Strukturbildung* is also gratefully acknowledged.

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## Figure captions

Figure 1.

The interaction potential (energy)  $U$ , as given by Eq. (1) [scaled by  $|Q_1Q_2|/\lambda_D$ ], is depicted vs. the reduced space variable  $x = r/\lambda_D$ . Here,  $s = +1$  (equal-sign grain charges) and  $\delta$  equals, respectively, 0 (1), in account for simple (dressed) Debye interactions, in the upper (lower) curves.

Figure 2.

The TDLW characteristic constant  $\omega_{T,0}^2$ , as given by Eq. (4) [scaled by  $|Q_1Q_2|/(M\lambda_D^3)$ ], is depicted vs. the lattice parameter  $\kappa = r_0/\lambda_D$ . Here,  $s = +1$  (equal-sign grain charges) and  $\delta$  equals, respectively, 0 (1), in account for simple (dressed) Debye interactions, in the upper (lower) curves.

Figure 3.

The TDLW dispersion curve: the square frequency  $\omega_T^2$ , as given by Eq. (2) (scaled by  $\omega_g^2$ ), is depicted vs. the wavenumber  $k$  (scaled by  $r_0^{-1}$ ), for arbitrary values of all parameters except  $s$  (here  $s = +1$ , for equal-sign grain charges) and  $\delta$ . The lower (upper) curves, respectively correspond to simple (dressed) Debye interactions, viz.  $\delta = 0$  (1).

Figure 4.

The LDLW characteristic constant  $\omega_{L,0}^2$ , as given by Eq. (7) [scaled by  $|Q_1Q_2|/(M\lambda_D^3)$ ], is depicted vs. the lattice parameter  $\kappa = r_0/\lambda_D$ . Here,  $s = +1$  (equal-sign grain charges) and  $\delta$  equals, respectively, 0 (1), in account for simple (dressed) Debye interactions, in the upper (lower) curves.

Figure 5.

The total potential  $U_{tot}(x) = U_{n-1,n}(r_0 + x) + U_{n+1,n}(r_0 - x)$  [here scaled by  $|Q_1Q_2|/\lambda_D$ ], which is felt by a dust grain in a alternating charge-sign bi-lattice, is depicted vs. the reduced position (displacement) variable  $x/\lambda_D$ . Here,  $s = -1$  (opposite-sign grain charges) and  $\delta = 1$  (dressed Debye interactions). The lattice parameter  $\kappa$  is equal to 6, 5, 4, 3, respectively, in the curves I, II, III, IV.

Figure 6.

(Switching off the dressing effect:) The total potential  $U_{tot}(x) = U_{n-1,n}(r_0 + x) + U_{n+1,n}(r_0 - x)$  [here scaled by  $|Q_1 Q_2|/\lambda_D$ ], which is felt by a dust grain in a alternating charge-sign bi-lattice, is depicted vs. the reduced position (displacement) variable  $x/\lambda_D$ . Here,  $s = -1$  (opposite-sign grain charges) and  $\delta = 0$  (*unperturbed* Debye interactions). The lattice parameter  $\kappa$  is equal to 6, 5, 4, 3, respectively, in the curves I, II, III, IV.

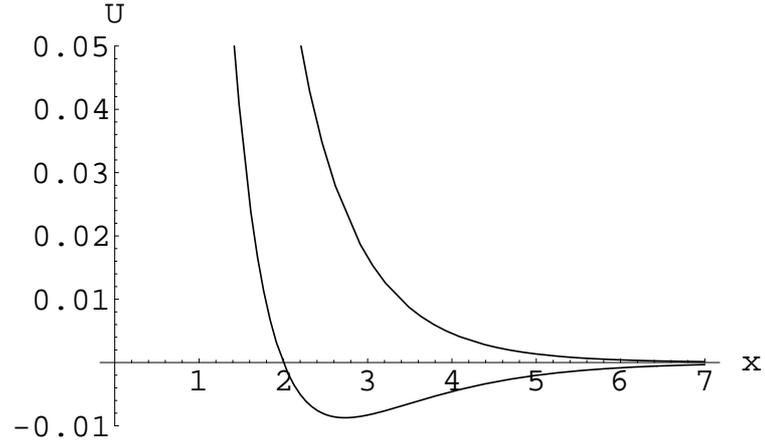


FIG. 1: The interaction potential (energy)  $U$ , as given by Eq. (1) [scaled by  $|Q_1 Q_2|/\lambda_D$ ], is depicted vs. the reduced space variable  $x = r/\lambda_D$ . Here,  $s = +1$  (equal-sign grain charges) and  $\delta$  equals, respectively, 0 (1), in account for simple (dressed) Debye interactions, in the upper (lower) curves.

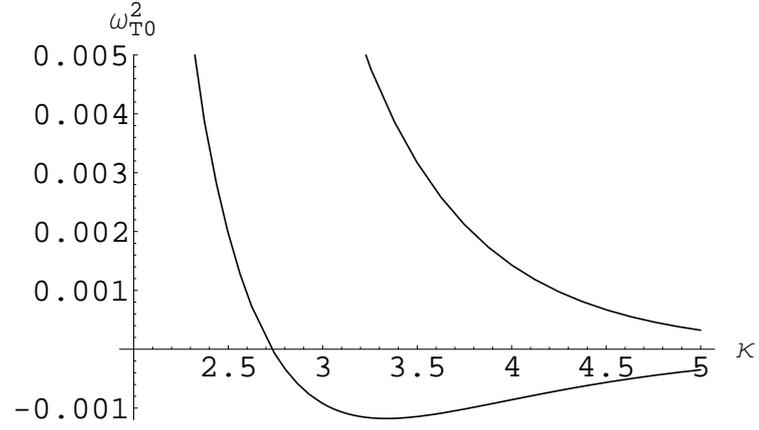


FIG. 2: The TDLW characteristic constant  $\omega_{T,0}^2$ , as given by Eq. (4) [scaled by  $|Q_1 Q_2|/(M\lambda_D^3)$ ], is depicted vs. the lattice parameter  $\kappa = r_0/\lambda_D$ . Here,  $s = +1$  (equal-sign grain charges) and  $\delta$  equals, respectively, 0 (1), in account for simple (dressed) Debye interactions, in the upper (lower) curves.

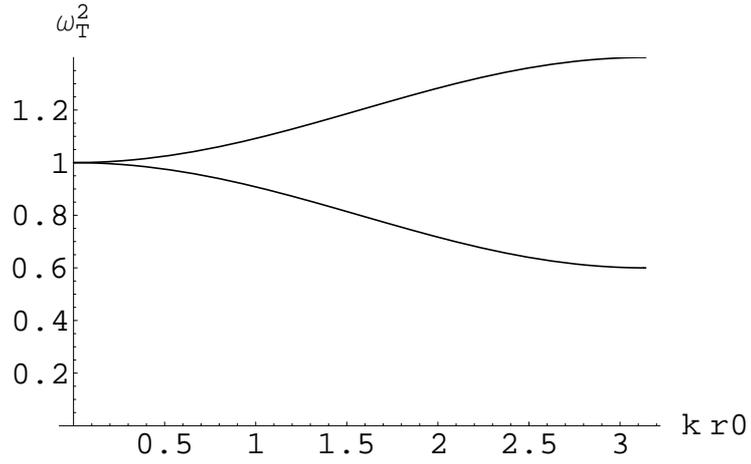


FIG. 3: The TDLW dispersion curve: the square frequency  $\omega_T^2$ , as given by Eq. (2) (scaled by  $\omega_g^2$ ), is depicted vs. the wavenumber  $k$  (scaled by  $r_0^{-1}$ ), for arbitrary values of all parameters except  $s$  (here  $s = +1$ , for equal-sign grain charges) and  $\delta$ . The lower (upper) curves, respectively correspond to simple (dressed) Debye interactions, viz.  $\delta = 0$  (1).

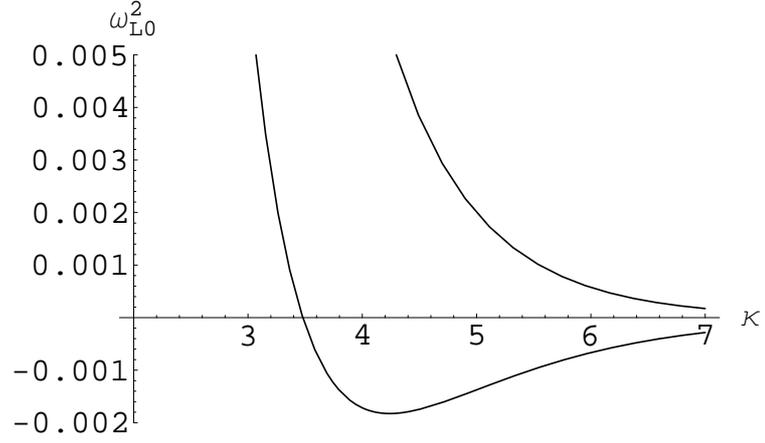


FIG. 4: The LDLW characteristic constant  $\omega_{L,0}^2$ , as given by Eq. (7) [scaled by  $|Q_1 Q_2|/(M\lambda_D^3)$ ], is depicted vs. the lattice parameter  $\kappa = r_0/\lambda_D$ . Here,  $s = +1$  (equal-sign grain charges) and  $\delta$  equals, respectively, 0 (1), in account for simple (dressed) Debye interactions, in the upper (lower) curves.

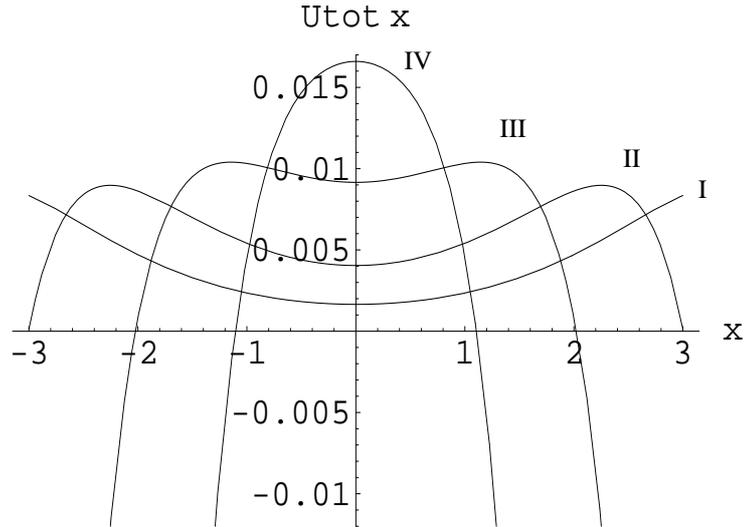


FIG. 5: The total potential  $U_{tot}(x) = U_{n-1,n}(r_0 + x) + U_{n+1,n}(r_0 - x)$  [here scaled by  $|Q_1 Q_2|/\lambda_D$ ], which is felt by a dust grain in an alternating charge-sign bi-lattice, is depicted vs. the reduced position (displacement) variable  $x/\lambda_D$ . Here,  $s = -1$  (opposite-sign grain charges) and  $\delta = 1$  (dressed Debye interactions). The lattice parameter  $\kappa$  is equal to 6, 5, 4, 3, respectively, in the curves I, II, III, IV.

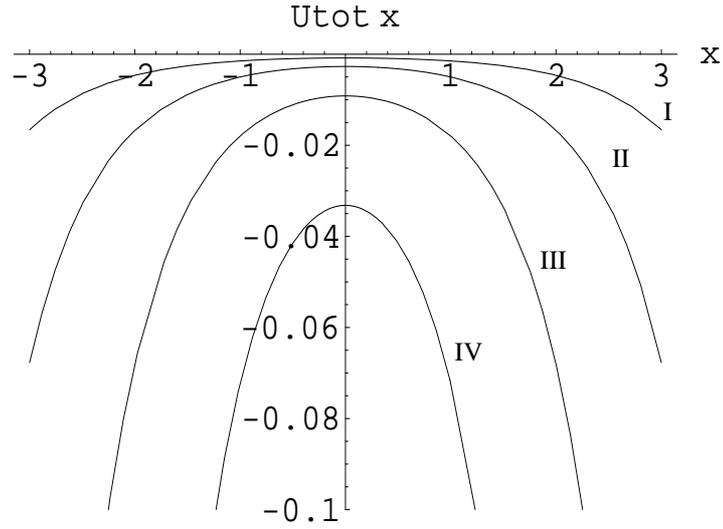


FIG. 6: (Switching off the dressing effect:) The total potential  $U_{tot}(x) = U_{n-1,n}(r_0+x) + U_{n+1,n}(r_0-x)$  [here scaled by  $|Q_1Q_2|/\lambda_D$ ], which is felt by a dust grain in a alternating charge-sign bi-lattice, is depicted vs. the reduced position (displacement) variable  $x/\lambda_D$ . Here,  $s = -1$  (opposite-sign grain charges) and  $\delta = 0$  (*unperturbed* Debye interactions). The lattice parameter  $\kappa$  is equal to 6, 5, 4, 3, respectively, in the curves I, II, III, IV. No stable equilibrium point occurs in this case.