Comment on "Dynamics in a Multicomponent Plasma near the Low-Frequency Cutoff"

In a recent Letter [1], Ganguli and Rudakov claim to have found new distinctive features of a multicomponent cold dusty plasma. However, the Hall-MHD behavior of a cold dusty plasma reveals that the results of Ref. [1] have severe limitations. To demonstrate this, we consider a cold plasma with mobile electrons and ions, as well as immobile negatively charged dust grains. At equilibrium, we have $n_{i0} - n_{e0} - Z_d n_{d0} = 0$, where n_{i0}, n_{e0}, and n_{d0} are the equilibrium ion, electron, and dust number densities, and Z_d is the charge state of the dust. The dynamics of low frequency (in comparison with the electron gyrofrequency), low phase velocity (in comparison with the speed of light c), long wavelength (in comparison with the electron skin depth and the ion gyroradius) electromagnetic waves in a uniform cold magnetoplasma is then governed by the ion continuity equation $\partial_t n_i + \nabla \cdot (n_i \mathbf{v}_i) = 0$, the ion momentum equation $(\partial_t + \mathbf{v}_i \cdot \nabla)\mathbf{v}_i = -(Z_d e n_d / n_e m_i c)\mathbf{v}_i \times \mathbf{B} - \mathbf{B} \times (\nabla \times \mathbf{B})/4\pi n_e m_i$, and Faraday's law $\partial_t \mathbf{B} = \nabla \times [(n_i \mathbf{v}_i \times \mathbf{B}/n_e) + \mathbf{v}_\mathrm{H} \times \mathbf{B}]$, which were presented for the first time in Ref. [2]. Here, n_e (n_i) is the electron (ion) number density, e is the magnitude of the electron charge, m_i is the ion mass, and $\mathbf{v}_\mathrm{H} = -(c/4\pi e n_e)\nabla \times \mathbf{B}$ is the Hall velocity. We stress that Eq. (4) of Ref. [1] is incomplete, since it does not contain the $\nabla \times (\mathbf{v}_\mathrm{H} \times \mathbf{B})$ term above. The latter is very important as it introduces a scale size of the order of the modified ion skin depth in the dust Hall-MHD plasma. By linearizing the equations above, we obtain a new dispersion relation

$$(\omega^2 - k_z^2 V_\mathrm{A}^2)(\omega^2 - \omega_\mathrm{A}^2) = \omega^2 \omega_\mathrm{A}^2 b + \Omega_\mathrm{R}^2(\omega^2 - \omega_\mathrm{A}^2 b) + 2\alpha \Omega_\mathrm{R} \omega_{ci} \omega_\mathrm{A}^2 b, \qquad (1)$$

where ω and $k = (k_\perp^2 + k_z^2)^{1/2}$ are the wave frequency and wave number, respectively, $\Omega_\mathrm{R} = Z_d n_{d0} \omega_{ci}/n_{e0}$ is the Rao cutoff frequency, $\omega_{ci} = eB_0/m_i c$ is the ion gyrofrequency, $V_\mathrm{A} = \alpha B_0/\sqrt{4\pi n_{i0} m_i}$ is the modified Alfvén speed, $\alpha = n_{i0}/n_{e0}$, $\omega_\mathrm{A} = kV_\mathrm{A}$, and $b = k_z^2 V_\mathrm{A}^2/\alpha^2 \omega_{ci}^2$. The terms involving b represent the contributions from the Hall current. Two comments are now in order. First, the perpendicularly propagating modified Alfvén-Rao (MAR) mode [2], $\omega = (\Omega_\mathrm{R}^2 + k_\perp^2 V_\mathrm{A}^2)^{1/2} \equiv \Omega_\mathrm{AR}$, is obtained from Eq. (1) above in the limit $k_z = 0$. Second, Eq. (8), $\omega^2 = \Omega_\mathrm{R}^2 + (k^2 + k_z^2)V_\mathrm{A}^2$, of Ref. [1] is obtained from our Eq. (1) above in the limits $\omega_\mathrm{A}^2 b \ll \Omega_\mathrm{R}^2, \omega^2\Omega_\mathrm{R}/2\alpha\omega_{ci}, \omega^2$ and $k_z V_\mathrm{A} \omega_\mathrm{A} \ll \omega^2$, whereas Eq. (9), $\omega^2 = k_z^2 k^2 V_\mathrm{A}^4/\Omega_\mathrm{R}^2$, of Ref. [1] is obtained from our Eq. (1) above in the limits $\omega_\mathrm{A}^2 b \ll \Omega_\mathrm{R}^2, \omega^2\Omega_\mathrm{R}/2\alpha\omega_{ci}, \omega^2$ and $\omega^2 \ll k_z V_\mathrm{A} \omega_\mathrm{A}$. Thus, there are severe limitations on the existence of these solutions, in contrast to the unnecessary inequalities $Z_d n_d/n_e \ll 1$ and $\Omega_\mathrm{R} \ll \omega_{ci}$ mentioned in Ref. [1].

Finally, we note the drawbacks of the energy and the ponderomotive force associated with the MAR modes (or the so-called rotational waves [1] in the dipole approximation). The perpendicular component of the ion fluid velocity associated with the MAR mode is $\mathbf{v}_{i\perp} = [\omega V_\mathrm{A}^2/B_0(\omega^2 - \Omega_\mathrm{R}^2)][\mathbf{k}_\perp + i(\Omega_\mathrm{R}/\omega)\hat{\mathbf{z}} \times \mathbf{k}_\perp]B_{1z}$. The unphysical singularity at $\omega = \Omega_\mathrm{R}$ in the ion kinetic energy density $KE = (1/2)m_i n_{i0}|\mathbf{v}_{i\perp}|^2$ can here be removed by using the linear dispersion relation $\omega = \Omega_\mathrm{AR}$ of the propagating MAR modes, yielding $KE = m_i n_{i0}(\Omega_\mathrm{AR}^2/2k_\perp^4 B_0^2)[k_\perp^2 + (\Omega_\mathrm{R}^2/\Omega_\mathrm{AR}^2)|\hat{\mathbf{z}} \times \mathbf{k}_\perp|^2]B_{1z}^2$. Furthermore, the electron and ion ponderomotive forces associated with the MAR modes are $m_e n_{e0}\langle(\mathbf{v}_e \cdot \nabla)\mathbf{v}_e\rangle + (n_{e0}e/c)\langle\mathbf{v}_e \times \mathbf{B}_1\rangle$ and $m_i n_{i0} < (\mathbf{v}_i \cdot \nabla)\mathbf{v}_i - (n_{i0}e/c)\langle\mathbf{v}_i \times \mathbf{B}_1\rangle$, respectively, where $\mathbf{v}_e \approx (n_{i0}/n_{e0})\mathbf{v}_i$. It then appears that the ion ponderomotive force is more dominant than the electron ponderomotive force in setting up the space charge electric field $\mathbf{E}_s \approx (n_{i0}/n_{d0}Z_d e)[(m_i/2)\nabla|\mathbf{v}_i|^2 - m_i\langle\mathbf{v}_i \times (\nabla \times \mathbf{v}_i)\rangle + (1/8\pi n_{i0})\nabla|\mathbf{B}_1|^2 - (1/4\pi n_{i0}) \times \langle(\mathbf{B}_1 \cdot \nabla)\mathbf{B}_1\rangle\rangle]$, which is responsible for the nonthermal ultralow frequency (in comparison with the dust gyrofrequency) dust Alfvén perturbations modulating the MAR modes. Finally, we note that Eq. (12) in Ref. [1] accounts only for the dust density perturbations in calculating the nonlinear frequency shift $\delta\Omega$ [e.g., Eq. (14)], without taking into account the contributions from the changes in the compressional magnetic field perturbations, which give rise to a nonlinear term involving the dispersion [the second term in the right-hand side of our ion momentum equation above] of propagating MAR modes. Hence, the nonlinear Schrödinger Eq. (15) of Ref. [1] is not reliable, and conclusions based on that equation are, therefore, not useful.

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