Low-frequency electromagnetic waves in a Hall-magnetohydrodynamic plasma with charged dust macroparticles

P. K. Shukla and I. Kourakis
Institut für Theoretische Physik IV and Centre for Plasma Science and Astrophysics, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany

L. Stenflo
Department of Physics, Umeå University, SE-90187 Umeå, Sweden

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A linear theory for intermediate-frequency [much smaller (larger) than the electron gyrofrequency (dust plasma and dust gyrofrequencies)], long wavelength (in comparison with the ion gyroradius and the electron skin depth) electromagnetic waves in a multicomponent, homogeneous electron-ion-dust magnetoplasma is presented. For this purpose, the generalized Hall-magnetohydrodynamic (GH-MHD) equations are derived for the case with immobile charged dust macroparticles. The GH-MHD equations in a quasineutral plasma consist of the ion continuity equation, the generalized ion momentum equation, and Faraday's law with the Hall term. The GH-MHD equations are Fourier transformed and combined to obtain a general dispersion relation. The latter is analyzed to understand the influence of immobile charged dust grains on various electromagnetic wave modes in a magnetized dusty plasma. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1842132]

The Alfvén wave[1] is classic in magnetized electron-ion plasmas. Within the framework of the ideal magnetohydrodynamic (MHD) theory, it is governed by the continuity and momentum equations for the plasma mass flow, together with Faraday's law, in which the electric field and the mass flow velocity are related by Ohm's law. In the Alfvén wave, the magnetic pressure provides the restoring force and the ion mass acts as inertia for the waves. The dispersion[2] of the Alfvén wave comes from finite frequency $\omega(<\omega_{ci}$, where ω_{ci} is the ion gyrofrequency), finite ion Larmor radius, finite ion polarization, and finite electron inertia effects. In dispersive Alfvén waves, the frozen-in field lines are broken, and there appears a linear coupling between various modes[3–8] (e.g., the Alfvén wave, the magnetosonic mode, the shear Alfvén wave, and the whistler). The dynamics of the dispersive Alfvén waves within the fluid model is governed by the Hall-MHD equations,[5] in which one uses the generalized Ohm's law to include the $\mathbf{J}\times\mathbf{B}$ force, where $\mathbf{J}$ is the plasma current and $\mathbf{B}$ is the total magnetic field in the plasma.

About a decade ago, Shukla[9] and Rao[10] reported the existence of a new cut-off frequency for circularly polarized electromagnetic ion-cyclotron Alfvén waves and for magnetosonic waves in an electron-ion plasma with stationary charged dust grains. Thus, the modification of the wave spectrum occurs due to the overall quasineutrality condition, as in dusty plasmas with negatively charged dust grains the ion number density is larger than the electron number density. Laboratory experiments[11] in a magnetized dusty plasma have revealed that static charged dust grains modify the electrostatic ion-cyclotron wave spectrum,[12] as well as the threshold criteria for its excitation. Here, the time scales are much shorter than the dust plasma and dust gyroperiods. On the other hand, the consideration of the dust particle dynamics also provides possibilities for new wave modes,[10,13–21] including dust shear Alfvén waves, dust magnetosonic waves, and dust whistlers, in which the inertia comes from the dust mass, and the wave frequencies could be of the order of or smaller than the dust gyrofrequency. Such ultra-low-frequency (ULF) electromagnetic waves can play a role in the formation of long wavelength Mach cones in Saturn's rings.[21] They may also account for the ULF fluctuations in low-temperature astrophysical objects (viz., molecular clouds) and in cometary tails.[22]

In this Brief Communication, we present the linear dispersion properties of intermediate-frequency (much smaller than the electron gyrofrequency, but much larger than the dust plasma and dust gyrofrequencies), long wavelength (in comparison with the ion gyroradius and the electron skin depth) electromagnetic waves in a multicomponent warm dusty magnetoplasma whose constituents are electrons, ions, and immobile charged dust macroparticles. We combine the inertialess electron equation of motion with the ion momentum equation and the equations of Faraday and Ampére to obtain a generalized ion momentum equation and Faraday's law with the Hall term. The ion continuity equation and the quasineutrality condition close the system of equations in the form of generalized Hall-MHD equations. The latter shows that the presence of immobile charged dust macroparticles greatly affects the ion dynamics in that it produces rotational motion of the ions[10] in the wave magnetic field. Following Ohsaki and Mahajan,[7] the present governing equations for our generalized Hall-magnetohydrodynamic (GH-MHD) plasma are then linearized, Fourier transformed, and combined to obtain a new dispersion relation, which exhibits a linear coupling between the Alfvén wave, the modified magnetosonic mode, the electromagnetic ion-cyclotron waves, and the whistlers in a warm dusty magnetoplasma. The general dispersion relation is analyzed in various limiting cases and its connection to previous works is established. The present results may be used to understand the dispersion properties of intermediate-frequency electromagnetic waves in magnetized laboratory and space plasmas.

Let us consider a three-component fully ionized dusty plasma composed of electrons, ions, and immobile charged dust particulates, with masses m_e, m_i, and m_d and charges $q_e=-e$, $q_i=Z_ie$, and $q_d=-Z_de$, where e is the magnitude of the electron charge, Z_i is the ion charge state, and Z_d is the number of electrons residing on a dust grain. The mass of micron-sized dust particles in laboratory and cosmic plasmas[19] is typically billion times larger than the ion mass. Our uniform plasma is supposed to be immersed in a homogeneous magnetic field $\mathbf{B_0}=B_0\hat{\mathbf{z}}$, where B_0 is the strength of the magnetic field and $\hat{\mathbf{z}}$ is the unit vector along the z axis in a Cartesian coordinate system.

We adopt the MHD system of equations for the electrons and ions. The massive dust particles are considered to be practically immobile (i.e., the dust density n_d is constant), since we are interested in examining the dispersion properties of electromagnetic waves on time scales much larger than the electron gyroperiod, but much shorter than the dust plasma and dust gyroperiods, viz., $\omega_{p,d}^{-1}$ and $\omega_{cd}^{-1}=m_dc/Z_deB_0$, respectively, where c is the speed of light in vacuum. We shall focus on long wavelengths (in comparison with the ion gyroradius and the electron skin depth) so that we can use the hydrodynamic equations for inertialess electrons and inertial ions. The electron and ion number densities $n_{e,i}$ and velocities $\mathbf{u}_{e,i}$ are governed by the continuity and momentum equations

$$\partial n_e/\partial t + \nabla \cdot (n_e\mathbf{u}_e) = 0, \tag{1}$$

$$\partial n_i/\partial t + \nabla \cdot (n_i\mathbf{u}_i) = 0, \tag{2}$$

$$0 = -e\left[\mathbf{E} + (1/c)\mathbf{u}_e \times \mathbf{B}\right] - (1/n_e)\nabla P_e, \tag{3}$$

and

$$m_iD_i\mathbf{u}_i = Z_ie\left[\mathbf{E} + (1/c)\mathbf{u}_i \times \mathbf{B}\right] - (1/n_i)\nabla P_i, \tag{4}$$

where we have ignored the electron inertia as the wave frequencies are much smaller than the electron gyrofrequency $\omega_{ce}=eB_0/m_ec$. In Eq. (4) $D_i\equiv(\partial/\partial t)+\mathbf{u}_i\cdot\nabla$. The pressures $P_{e,i}$ are assumed to obey $P_{e,i}\sim n_{e,i}^{\gamma_{e,i}}$, where γ represents the adiabatic index, i.e., $\gamma=3$ for adiabatic compression, $\gamma=5/3$ in three dimensions, and $\gamma=1$ for isothermal compression. Accordingly, we will set $\nabla P_{e,i}=\gamma_{e,i}T_{e,i}\nabla n_{e,i}$, where $T_{e,i}$ is the temperature in energy units. Furthermore, $\mathbf{E}$ is the wave electric field and $\mathbf{B}$ is the sum of the static and wave magnetic fields, viz., $\mathbf{B}=\mathbf{B}_0+\mathbf{b}$. The system is closed with the Maxwell equations. Neglecting the displacement current, Ampère's law reads

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} \equiv \frac{4\pi}{c}\sum_\alpha q_\alpha n_\alpha \mathbf{u}_\alpha = \frac{4\pi e}{c}(Z_in_i\mathbf{u}_i - n_e\mathbf{u}_e), \tag{5}$$

and Faraday's law is

$$\nabla \times \mathbf{E} = -(1/c)(\partial\mathbf{B}/\partial t). \tag{6}$$

We note that Eq. (5) is valid for low phase speed (in comparison with c) electromagnetic waves.

At equilibrium, the overall neutrality condition is

$$n_{e,0} - Z_in_{i,0} + Z_dn_d = 0, \tag{7}$$

where the subscript 0 denotes the unperturbed quantity.

By using Eq. (3) we can eliminate $\mathbf{E}$ from Eq. (4) to obtain

$$m_in_iD_i\mathbf{u}_i = n_iZ_i(e/c)(\mathbf{u}_i - \mathbf{u}_e) \times \mathbf{B} - \gamma_iT_i\nabla n_i - (\gamma_eZ_in_{i0}/n_{e0})T_e\nabla n_e, \tag{8}$$

which may be combined with (5), viz.,

$$\mathbf{u}_e = Z_i(n_i/n_e)\mathbf{u}_i - (c/4\pi en_e)(\nabla \times \mathbf{B}), \tag{9}$$

in order to eliminate $\mathbf{u}_e$ in Eq. (8). The electric field $\mathbf{E}$ in Eq. (6) can then be eliminated by means of Eqs. (3) and (9) to obtain

$$\partial\mathbf{B}/\partial t = \nabla \times \left[(Z_in_i/n_e)(\mathbf{u}_i \times \mathbf{B})\right] - (c/4\pi e)\nabla \times \left[(1/n_e)(\nabla \times \mathbf{B}) \times \mathbf{B}\right]. \tag{10}$$

Equations (2), (8) and (10) and the quasineutrality condition for the perturbed density constitute a system of equations for studying waves in dust Hall-MHD plasmas.

Letting $n_i\approx n_{i,0}+n_1$ and $\mathbf{u}_i=\mathbf{0}+\mathbf{v}$, where $n_1\ll n_{i,0}$ is a small perturbation in the density, we have the ion continuity equation

$$\partial n_1/\partial t + n_{i,0}\nabla \cdot \mathbf{v} = 0, \tag{11}$$

the ion momentum equation

$$\begin{aligned}\partial\mathbf{v}/\partial t &= -(Z_iZ_den_d/n_{e,0}m_ic)(\mathbf{v} \times \mathbf{B_0}) + (Z_i/4\pi n_{e,0}m_i)(\nabla \times \mathbf{b}) \times \mathbf{B_0} - (c_s^2/n_{i,0})\nabla n_1 = -\Omega_R(\mathbf{v} \times \hat{\mathbf{z}}) \\ &\quad + (\alpha B_0/4\pi n_{i,0}m_i)(\nabla \times \mathbf{b}) \times \hat{\mathbf{z}} - (c_s^2/n_{i,0})\nabla n_1,\end{aligned} \tag{12}$$

and the magnetic field evolution equation

$$\begin{aligned}\partial\mathbf{b}/\partial t &= (Z_in_{i,0}/n_{e,0})\nabla \times (\mathbf{v} \times \mathbf{B_0}) - (c/4\pi en_{e,0})\nabla \times [(\nabla \times \mathbf{b}) \times \mathbf{B_0}] = \alpha B_0\nabla \times (\mathbf{v} \times \hat{\mathbf{z}}) \\ &\quad - (\alpha cB_0/4\pi Z_ien_{i,0})\nabla \times [(\nabla \times \mathbf{b}) \times \hat{\mathbf{z}}],\end{aligned} \tag{13}$$

where we have used the quasineutrality condition, given by Eq. (7), with $Z_d>0$ ($Z_d<0$) for negatively (positively) charged dust grains, and introduced the notations $\Omega_R=Z_dn_d\omega_{ci}/n_{e,0}$ and $\alpha=Z_in_{i,0}/n_{e,0}$, where $\omega_{ci}=Z_ieB_0/m_ic$ is the ion gyrofrequency. Furthermore, we have denoted the modified ion sound speed by $c_s=[(\gamma_in_{i,0}T_i+\gamma_eZ_i^2n_{i,0}^2T_e/n_{e,0})/m_in_{i,0}]^{1/2}$.

Equations (11)–(13) form a closed system which describes the evolution of small perturbations of the ion density, the ion velocity, and the magnetic field in our dust Hall-MHD plasma. In a dust free plasma with $\alpha=1$ or $n_{e,0}=Z_in_{i0}$, the ion rotation frequency Ω_R vanishes and Eq. (12) without the Ω_R dictates acceleration of ions by the $\mathbf{J}\times\mathbf{B}_0$ and $\nabla(P_{e1}+P_{i1})$ forces, while Eq. (13) with $\alpha=1$ simply depicts the evolution of the wave magnetic field in the presence of a nonsolenoidal electric field $\mathbf{E}=-\mathbf{v}_e\times\mathbf{B}_0$. In a dusty Hall-MHD plasma with negatively charged dust grains, we have $\alpha>1$, and correspondingly there appears enhanced charge separation due to the wave electric field. The resulting enhanced electron fluid velocity produces a new Lorentz centripetal force [the first term in the right-hand side of Eq. (12)], which in combination with the $\mathbf{J}\times\mathbf{B}_0$ and pressure gradient forces produces rotation of the ions around the negatively charged static dust grains. The rotational force acting on the ions is then responsible for a non trivial coupling between various wave modes in dusty plasmas, while due to $\alpha>1$, we have increased Alfvén wave phase speed and ion skin depth.

Let us now consider small amplitude propagating electromagnetic waves around the equilibrium state $\{n_{i,0}, \mathbf{0}, \mathbf{B_0}\}$ (where the perturbations are $\{n_1, \mathbf{v}, \mathbf{b}\}$). Following Ref. 7, we thus linearize our governing equations to derive a fairly general dispersion relation for the wave propagation in our uniform dusty magnetoplasma. By letting $\partial/\partial t\rightarrow -i\omega$ and $\nabla\rightarrow i\mathbf{k}$, where ω and $\mathbf{k}=\mathbf{k}_\perp+\hat{\mathbf{z}}k_z$ denote the wave frequency and the wave vector, respectively, we then obtain from Eqs. (11)–(13)

$$\omega n_1 - n_{i,0}\mathbf{k}\cdot\mathbf{v}=0, \tag{14}$$

$$\omega\mathbf{v}=-i\Omega_R(\mathbf{v}\times\hat{\mathbf{z}})-\frac{\alpha B_0}{4\pi m_i n_{i,0}}(k_z\mathbf{b}-b_z\mathbf{k})+(\mathbf{k}\cdot\mathbf{v})\mathbf{k}\frac{c_s^2}{\omega}, \tag{15}$$

and

$$\omega\mathbf{b}=-\alpha B_0[k_z\mathbf{v}-(\mathbf{k}\cdot\mathbf{v})\hat{\mathbf{z}}]+i\frac{\alpha c B_0}{4\pi e n_{i,0}Z_i}k_z(\mathbf{k}\times\mathbf{b}). \tag{16}$$

Using the constraint $\nabla\cdot\mathbf{B}=0$, i.e., $\mathbf{k}\cdot\mathbf{b}=0$, the inner product of $\mathbf{k}$ with Eq. (15) gives

$$\mathbf{k}\cdot\mathbf{v}=\frac{\omega}{\omega^2-k^2c_s^2}\left[\frac{\alpha B_0}{4\pi m_i n_{i,0}}k^2 b_z - i\Omega_R(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}}\right], \tag{17}$$

which, combined with the z components of Eqs. (15) and (16), yields

$$\omega b_z=\frac{(\omega^2-k_z^2c_s^2)k^2V_A^2}{\omega(\omega^2-k^2c_s^2)}b_z+i\frac{\alpha c B_0}{4\pi Z_i e n_{i,0}}k_z j_z - i\alpha B_0\Omega_R\frac{\omega^2-k_z^2c_s^2}{\omega(\omega^2-k^2c_s^2)}(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}}, \tag{18}$$

where we defined $j_z=(\mathbf{k}\times\mathbf{b})\cdot\hat{\mathbf{z}}$, as well as the (dust-modified) Alfvén speed $V_A=\alpha B_0/\sqrt{4\pi m_i n_{i,0}}\equiv\alpha v_A$, where $\alpha=Z_i n_{i,0}/n_{e,0}$.

From Eqs. (15) and (16), one also obtains

$$\omega j_z=-\alpha B_0 k_z(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}}-i\frac{\alpha c B_0}{4\pi Z_i e n_{i,0}}k^2k_z b_z \tag{19}$$

and

$$\omega\left[1-\frac{\Omega_R^2(\omega^2-k_z^2c_s^2)}{\omega^2(\omega^2-k^2c_s^2)}\right](\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}} =-\frac{\alpha B_0}{4\pi m_i n_{i,0}}k_z j_z+i\frac{\omega\Omega_R(\omega^2-k_z^2c_s^2)}{\omega^2(\omega^2-k^2c_s^2)}\frac{\alpha B_0}{4\pi m_i n_{i,0}}k^2 b_z. \tag{20}$$

Equations (18)–(20) form a closed system in terms of b_z, j_z, and $(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}}$. The latter two equations may now be solved, e.g., in terms of the latter two variables, and the solutions may then be inserted into Eq. (18).

Combining Eqs. (18)–(20) we obtain

$$j_z=-i\left[\omega^2-\frac{\Omega_R^2(\omega^2-k_z^2c_s^2)}{(\omega^2-k^2c_s^2)}-k_z^2V_A^2\right]^{-1}\times\left\{\left[1-\frac{\Omega_R^2(\omega^2-k_z^2c_s^2)}{\omega^2(\omega^2-k^2c_s^2)}\right]\frac{1}{\alpha\omega_{ci}}+\frac{\Omega_R(\omega^2-k_z^2c_s^2)}{\omega^2(\omega^2-k^2c_s^2)}\right\}\times k_z k^2V_A^2\omega b_z \tag{21}$$

and

$$(\mathbf{k}\times\mathbf{v})\cdot\hat{\mathbf{z}}=i\left[\omega^2-\frac{\Omega_R^2(\omega^2-k_z^2c_s^2)}{(\omega^2-k^2c_s^2)}-k_z^2V_A^2\right]^{-1}\times\left[\frac{\Omega_R(\omega^2-k_z^2c_s^2)}{(\omega^2-k^2c_s^2)}+\frac{k_z^2V_A^2}{\alpha\omega_{ci}}\right]\frac{k^2V_A}{\sqrt{4\pi m_i n_{i,0}}}b_z. \tag{22}$$

Substituting Eqs. (21) and (22) into Eq. (18), we obtain a new dispersion relation

$$(\omega^2-k_z^2V_A^2)\left[\omega^2(\omega^2-k^2c_s^2)-(\omega^2-k_z^2c_s^2)k^2V_A^2\right] =\omega^2(\omega^2-k_z^2c_s^2)\Omega_R^2+\frac{k_z^2k^2V_A^4}{\alpha^2\omega_{ci}^2}\left[\omega^2(\omega^2-k^2c_s^2)-\Omega_R^2(\omega^2-k_z^2c_s^2)\right]+\frac{2k_z^2k^2V_A^4\Omega_R}{\alpha\omega_{ci}}(\omega^2-k_z^2c_s^2), \tag{23}$$

where $k^2=k_x^2+k_y^2+k_z^2\equiv k_\perp^2+k_z^2$. As a first remark, we note the modification due to the dust via Ω_R and α in $V_A=\alpha v_A$. Furthermore, we note that the last three terms in the right-hand side of Eq. (23) are due to the Hall-term in Eq. (10), and thus disappear in the *ideal* MHD limit (the first term in the right-hand side would then be the sole modification due to the stationary dust). The same terms also disappear in the purely perpendicular propagation limit (i.e., $k_z=0$).

In order to gain some insight, we now check the above results in the vanishing dust limit. With $\Omega_R=0$, $\alpha=1$, and $V_A=v_A$ in Eq. (23) we thus obtain

$$(\omega^2-k_z^2v_A^2)\left[\omega^4-k^2(v_A^2+c_s^2)\omega^2+k_z^2k^2c_s^2v_A^2\right] =(\omega^2-k^2c_s^2)(\omega^2k_z^2k^2v_A^4/\omega_{ci}^2) \tag{24}$$

or

$$(\omega^2-k_z^2v_A^2)\left(\omega^2-k^2v_A^2\frac{\omega^2-k_z^2c_s^2}{\omega^2-k^2c_s^2}\right)=\frac{\omega^2k_z^2k^2v_A^4}{\omega_{ci}^2}. \tag{25}$$

The dispersion relation (24) has been derived in Ref. 3 (see the Appendix A therein) and analyzed in Refs. 4 and 5. Equation (24) contains the electromagnetic ion-cyclotron Alfvén modes, the fast and slow magnetosonic modes, the kinetic Alfvén waves, and the long wavelength (in comparison with the electron skin depth) whistlers. For example, the magnetic field aligned dispersive electromagnetic ion-cyclotron Alfvén waves, $\omega = k_z v_A (1 \pm \omega/\omega_{ci})^{1/2}$, are obtained from Eq. (24) in the limit $c_s = 0$ and $k_\perp = 0$. Here + (−) corresponds to right- (left-) hand circularly polarized waves. Furthermore, the whistler frequency $\omega = k_z k c^2 \omega_{ce}/\omega_{pe}^2$ is obtained from Eq. (24) in the limits $\omega \gg k v_A$ and $c_s = 0$. The kinetic Alfvén waves $\omega \approx k_z v_A (1 + k_\perp^2 c_s^2/\omega_{ci}^2)^{1/2}$ are obtained from Eq. (24) in the limits $c_s \ll v_A$, $k_z c_s \ll \omega \ll \omega_{ci}$, $k_\perp v_A$, $k_\perp c_s$. On the other hand, in the case of propagation of waves perpendicular to the magnetic field direction, viz., $k_z = 0$ and $k = k_\perp \equiv (k_x^2 + k_y^2)^{1/2}$, one recovers from Eq. (25) the (fast) magnetosonic mode $\omega = k_\perp (v_A^2 + c_s^2)^{1/2}$.

In the case of wave propagation perpendicular to the magnetic field, viz., $k_z = 0$ and $k = k_\perp \equiv (k_x^2 + k_y^2)^{1/2}$, one recovers from Eq. (23), the modified magnetosonic mode

$$\omega^2 = \Omega_R^2 + k_\perp^2 (c_s^2 + V_A^2). \tag{26}$$

We note the frequency cutoff $\omega(k_\perp = 0) = \Omega_R$, which was derived by Rao[10] [cf. Eq. (26a) therein]. This cutoff is absent in a usual electron-ion (i.e., dust-free) two component plasma.

In the case of wave propagation along the magnetic field direction, i.e., for $k_x = k_y = k_\perp = 0$ and $k = k_z$, one easily obtains from Eq. (23)

$$(\omega^2 - k_z^2 V_A^2)^2 = \omega^2 \Omega_R^2 + \frac{k_z^4 V_A^4}{\alpha^2 \omega_{ci}^2} (\omega^2 - \Omega_R^2) + 2 k_z^4 V_A^4 \frac{\Omega_R}{\alpha \omega_{ci}}, \tag{27}$$

which can be exactly rewritten in the simpler form

$$k_z^2 v_A^2 = \frac{\omega^2 \omega_{ci}}{\omega_{ci} \pm \omega} \mp \frac{Z_d n_d}{Z_i n_{i,0}} \omega \omega_{ci}, \tag{28}$$

and which precisely agrees with the well known result[9] for the magnetic field aligned circularly polarized electromagnetic waves, even in a warm magnetoplasma.

In a cold dusty plasma ($c_s = 0$), the dispersion relation (23) reduces to

$$(\omega^2 - k_z^2 V_A^2)(\omega^2 - \omega_A^2) = \omega^2 \omega_A^2 b + \Omega_R^2 (\omega^2 - \omega_A^2 b) + 2\alpha \Omega_R \omega_{ci} \omega_A^2 b, \tag{29}$$

where $\omega_A = k V_A$ and $b = k_z^2 V_A^2/\alpha^2 \omega_{ci}^2$. In the limits $\omega_A^2 b \ll \Omega_R^2$, $\omega^2 \Omega_R / 2\alpha\omega_{ci}$, ω^2 and $k_z V_A \omega_A \ll \omega^2$, one obtains $\omega^2 = \Omega_R^2 + (k^2 + k_z^2) V_A^2$, whereas in the limits $\omega_A^2 b \ll \Omega_R^2$, $\omega^2 \Omega_R / 2\alpha\omega_{ci}$, ω^2 and $\omega^2 \ll k_z V_A \omega_A$, we have $\omega^2 = k_z^2 k^2 V_A^4 / \Omega_R^2$. Those limits have been recently deduced by Ganguli and Rudakov[23] by neglecting from the outset the electron and ion pressure terms in Eq. (8) and the Hall term in Eq. (10).

We have here presented an investigation of the linear propagation of intermediate-frequency ($\omega_{cd} \ll \omega \ll \omega_{ce}$), long wavelength (in comparison with the ion gyroradius and the electron skin depth) electromagnetic waves in a warm dusty Hall-MHD plasma that is uniform. Our dusty plasma is composed of electrons, ions, and immobile charged dust macroparticles. The presence of the latter is responsible for enhancing the electron flow velocity and thus increasing the charge separation by the wave electric field, as well as for enhancing the Alfvén speed and for producing the ion rotation around the stationary charged dust grains, which gives rise to the existence of the Rao cut-off frequency[10] (which is nonexisting in an ordinary electron-ion plasma). Our new dispersion relation reveals that stationary dust grains significantly modify the dispersion relation of coupled dust Alfvén-ion-cyclotron-modified magnetosonic waves-whistlers in a nontrivial manner. We have shown how the previously known results can be recovered as limiting cases of our general dispersion relation. The present results are a prerequisite to understand the dispersion properties of intermediate-frequency, long wavelength electromagnetic waves in laboratory and space magnetoplasmas where a significant amount of charged dust grains is present.

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