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Modulated whistler wave packets associated with density perturbations

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The parametric interaction between large amplitude whistlers and ponderomotively driven quasistationary density perturbations in plasmas is considered. A cubic nonlinear Schrödinger equation is derived and then solved analytically to show the occurrence of modulational instability as well as the existence of bright and dark envelope solitons, which are referred to as *whistlerons*. Explicit whistleron profiles are presented and the relevance to space and laboratory plasmas is discussed. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1814997]

I. INTRODUCTION

Long wavelength electron-cyclotron electromagnetic magnetic field-aligned plasma waves (*whistlers*), a common phenomenon in the Earth's magnetosphere, can be excited by a variety of linear instabilities, e.g., caused by electron temperature anisotropy or propagating beams.[1,2] They may also be excited, for example, when electromagnetic energy from a lightning strike enters a magnetic field line duct (a process which is more efficient near the magnetic poles). Such electromagnetic energy can be guided along closed magnetic field lines though the enhanced ionization usually present near such magnetic field ducts. The wave travels along the field line and can be observed at the opposite pole (conjugate point). Because the wave is highly dispersive (see above) different frequencies arrive at the conjugate point at different times and, using a radio receiver, a descending glide tone can be heard for each lightning strike occurring in the opposite hemisphere. Whistlers also occur widely in the plasmasphere, magnetosheath, and terrestrial foreshock.[2] Of particular interest in the following are whistler wave-related modulated structures associated with density perturbations, which have most often been observed by recent satellite missions (e.g., Cluster,[3] Freja[4]) in the magnetosphere as well as in laboratory experiments.[5–7] A comprehensive review of whistler-related phenomena in both space and experimental physics can be found in Ref. 8.

In a generic manner, as the wave amplitude increases, nonlinear effects become important. One such effect, which is for long known to govern wave propagation in dispersive media, is the nonlinear modulation of the carrier wave's amplitude due to parametric coupling with low-frequency copropagating wave modes. Following the work of Hasegawa,[9] who used a reductive perturbation method in order to study the modulation of electron-cyclotron waves by low-frequency magnetohydrodynamic (MHD) fluctuations, Karpman and Washimi[10] have included the ponderomotive force in a description of the modulation of electromagnetic waves with low-frequency magnetoacoustic waves. Those results were then applied by the same authors, to a study of the modulation of high-frequency magnetic field-aligned electron-cyclotron waves due to coupling with slow magnetosonic waves,[11] within a MHD approximation, and the formalism was later applied by Shukla and co-workers[12–14] in a description of the parametric interaction between whistlers and ion-acoustic waves.

In this paper, we are interested in studying the parametric interaction between large amplitude whistlers and ponderomotively driven nonlinear dispersive ion-acoustic perturbations, by generalizing the Karpman equations. A cubic time-dependent nonlinear Schrödinger is derived, describing the spatiotemporal behavior of the modulated whistler electric field wave packet. By solving the latter, we aim at pointing out the occurrence of modulational instability as well as the existence of bright and dark envelope solitons. Explicit profiles for these *whistlerons* are presented and the relevance of this study to space and laboratory plasmas will be discussed.

II. THE MODEL

We will assume a collisionless plasma consisting of ions (denoted by i; mass m_i, charge $q_i=+e$; e denotes the absolute value of the electron charge) and electrons (mass m_e, charge $-e$), which is embedded in a magnetic field $\mathbf{B}=B_0\hat{z}$, where B_0 is the magnetic field induction and $\hat{z}$ is the unit vector along the z axis. Overall charge neutrality is assumed at equilibrium.

Let us consider the nonlinear coupling between right-hand circularly polarized electron-cyclotron (whistler) waves, associated with an electric field $\mathbf{E}=E(\hat{x}+i\hat{y})\exp(i\mathbf{kr}-i\omega t)+\text{c.c.}$ (complex conjugate), and low-phase velocity [compared to the electron thermal speed $v_{th,e}=(T_e/m_e)^{1/2}$] ion-acoustic waves, propagating in our magnetized plasma. We will adopt a cold plasma approximation for whistlers, assuming that $|\omega-\omega_{c,e}|\gg kv_{th,e}$, where $\omega_{c,e}=eB_0/m_ec$ denotes the electron gyrofrequency.

The wave frequency ω and the wave number $\mathbf{k}=(\mathbf{k}_\perp,k_\parallel)$ are related by the relation:[1] $n^2=c^2k^2/\omega^2=1+\omega_{p,e}^2/[\omega(\omega_{c,e}\cos\theta-\omega)]$ [where the angle $\theta=\mathbf{k}\cdot\hat{z}/k$ mea-

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sures the propagation obliqueness with respect to the field; obviously, $k=(k_\perp^2+k_\parallel^2)^{1/2}$], where $\omega_{p,e}$ ($\omega_{p,i}$) denotes the electron (ion) plasma frequency $\omega_{p,\alpha}=(4\pi n_\alpha e^2/m_\alpha)^{1/2}$, $\alpha=e$ or i; the ion plasma frequency is much lower than the frequency of interest, i.e., $\omega \gg \omega_{p,i}$. Assuming a high medium refractive index $n=kc/\omega \gg 1$, one may approximate the electron cyclotron frequency by

$$\omega \approx \frac{c^2k^2\omega_{c,e}\cos\theta}{\omega_{p,e}^2+c^2k^2}. \tag{1}$$

Note that the well-known approximate whistler dispersion relation $\omega \approx c^2k^2\omega_{c,e}\cos\theta/\omega_{p,e}^2$ arises in the long wavelength limit ($k \ll \omega_{p,e}/c$). In the following, we will consider magnetic field-aligned propagation, i.e., $\mathbf{k}=k\hat{z}$ (or $\theta=0$).

The interaction between whistlers and ion-acoustic waves produces an electric field envelope which obeys a nonlinear Schrödinger-type equation:[10–13]

$$i\left(\frac{\partial E}{\partial t}+v_g\frac{\partial E}{\partial z}\right)+P\frac{\partial^2 E}{\partial z^2}-\Delta E=0, \tag{2}$$

where $v_g=\omega'(k)=2\omega(\omega_{c,e}-\omega)/k\omega_{c,e}$ is the group velocity of the electron-cyclotron waves and the group velocity dispersion coefficient $P=\omega''(k)/2=v_g(\omega_{c,e}-4\omega)/2k\omega_{c,e}$. Clearly, whistlers are characterized by a normal (anomalous) group dispersion, viz., $P>0$ ($P<0$), if $\omega<\omega_{c,e}/4$ ($\omega>\omega_{c,e}/4$, respectively). The nonlinear frequency shift Δ is given by[12,13]

$$\Delta=\frac{\omega\omega_{p,e}^2}{2(\omega-\omega_{c,e})}\frac{v_g}{kc^2}\left(N-\frac{2v}{v_g}\right), \tag{3}$$

where $N=\delta n_e/n_{e,0}$ denotes the electron density perturbation δn_e scaled over the equilibrium density $n_{e,0}$ (recall that $n_{e,0} \approx n_{i,0} \equiv n_0$ due to overall neutrality at equilibrium).

The dynamics of the low-frequency electron fluid density fluctuations obeys the (dimensionless) continuity equation $\partial N/\partial t+\partial v/\partial z=0$ (the electron streaming velocity v is assumed along the magnetic field) which in combination with the electron and ion momentum equations and Poisson's equation has been shown in Refs. 12–14 to provide the following evolution equation for the density perturbation:

$$\frac{\partial^2 N}{\partial t^2}-c_s^2\frac{\partial^2}{\partial z^2}\left(N+N^2+\lambda_{D,e}^2\frac{\partial^2 N}{\partial z^2}\right) = \frac{c_s^2\omega_{p,e}^2}{\omega(\omega-\omega_{c,e})}\left(\frac{\partial^2}{\partial z^2}+\frac{2}{v_g}\frac{\partial^2}{\partial z\partial t}\right)\frac{|E|^2}{4\pi n_0 T_{eff}}, \tag{4}$$

where $c_s=(T_{eff}/m_i)^{1/2}$ is the ion-acoustic speed, $\lambda_{D,e}=c_s/\omega_{p,i}$ is the electron Debye length, and $T_{eff}=\gamma_e T_e+\gamma_i T_i$, where T_α (γ_α) denotes the temperature (specific heat ratio, respectively) of the species $\alpha=e,i$.

In the following, we shall assume that $v \ll v_g$. Equations (2) and (4) then form a closed system, which describes the simultaneous evolution of the density perturbation N and the electric field amplitude E. One notices in the right-hand side of Eq. (4) the appearance of the ponderomotive force ($\sim|E|^2$) acting on the plasma slow motion. Some insight in the dynamics may be gained by assuming a very slow time variation (i.e., $\partial^2 N/c_s^2\partial t^2$, $\partial^2 N/v_g\partial z\partial t \ll \partial^2 N/\partial z^2$), and neglecting the nonlinearity ($\sim N^2$) and dispersive ($\sim\partial^4 N/\partial z^4$) terms. Assuming a vanishing density perturbation at infinity, one then obtains

$$N=\frac{\omega_{p,e}^2}{\omega(\omega_{c,e}-\omega)}\frac{1}{4\pi n_0 T}(|E|^2-|E_\infty|^2), \tag{5}$$

where the integration constant E_∞ denotes the field amplitude at infinity and $T=T_e+T_i$. Now, combining with Eq. (3) (for $v \ll v_g$) one obtains

$$\Delta \approx -\frac{\omega_{p,e}^4}{2(\omega-\omega_{c,e})^2}\frac{v_g}{kc^2}\frac{1}{4\pi n_0 T}(|E|^2-|E_\infty|^2) \equiv -Q(|E|^2-|E_\infty|^2). \tag{6}$$

The quantity Q, whose definition in obvious, turns out to be equal to $Q=(\omega_{p,e}^2/\omega_{c,e})1/(4\pi n_0 T)$. Equation (2) may now be cast in the reduced nonlinear Schrödinger form

$$i\frac{\partial \mathcal{E}}{\partial \tau}+p\frac{\partial^2\mathcal{E}}{\partial s^2}+q|\mathcal{E}|^2\mathcal{E}=0, \tag{7}$$

where $\mathcal{E}=E/\sqrt{4\pi n_0 T}$ denotes the (reduced) electric field amplitude, and time and space have been scaled over the electron plasma period and Debye length, respectively, i.e., $t \to \tau=\omega_{p,e}t$ and $z \to \zeta=z/\lambda_{D,e}$; a Galilean transformation to a frame moving at the group velocity, viz., $z'=z-v_g t$ [i.e., $s=\zeta-(v_g/v_{th,e})\tau$] was carried out, for simplicity. See that the constant contribution to the right-hand side of Eq. (5) (related to the value of E at infinity) was omitted in Eq. (7), since it simply leads to a linear phase shift in $\mathcal{E}$: it is thus eliminated upon simply setting $\mathcal{E} \to \mathcal{E}\exp(iq|\mathcal{E}_\infty|^2\tau)$.

III. ANALYTICAL AND NUMERICAL RESULTS

As far as the (now dimensionless) coefficients in Eq. (7) are concerned, one immediately sees that the nonlinearity coefficient $q=Q4\pi n_0 T/\omega_{p,e}=\omega_{p,e}/\omega_{c,e}$ in Eq. (7) is positive, while the dispersion coefficient $p=P/(\lambda_{D,e}^2\omega_{p,e})$, which is now given by

$$p=\frac{v_g(\omega_{c,e}-4\omega)}{2k\omega_{c,e}}\frac{1}{\lambda_{D,e}^2\omega_{p,e}}=\omega_{p,e}\omega_{c,e}\frac{c^2}{\lambda_{D,e}^2}\frac{\omega_{p,e}^2-3c^2k^2}{(\omega_{p,e}^2+c^2k^2)^3} \tag{8}$$

is positive (negative) for ω lower (higher) than $\omega_{c,e}/4$, i.e., for $ck<\omega_{p,e}/\sqrt{3}$ (or $ck>\omega_{p,e}/\sqrt{3}$, respectively), which amounts to $k\lambda_{D,e}<v_{th,e}/c\sqrt{3}$ (or $k\lambda_{D,e}>v_{th,e}/c\sqrt{3}$, respectively). The same goes for the sign of the coefficient ratio

$$\eta \equiv \frac{p}{q}=\left(1-\frac{\omega}{\omega_{c,e}}\right)^2\left(1-\frac{4\omega}{\omega_{c,e}}\right)\left(\frac{\omega_{c,e}}{\omega_{p,e}}\right)^2\frac{m_e c^2}{T_e}, \tag{9}$$

which is here defined for later reference.

According to the generic nonlinear Schrödinger (NLS) formalism, exposed in detail elsewhere (see, e.g., in Refs. 15–17), one expects the field envelope $\mathcal{E}$, which is associated with the whistlers, to be modulationally unstable if pq is positive, i.e., if $\omega<\omega_{c,e}/4$. To see this, one may first check that the nonlinear Schrödinger equation (NLSE) is satisfied by the plane wave solution $\mathcal{E}(s,\tau)=\mathcal{E}_0 e^{iq|\mathcal{E}_0|^2 T}+\text{c.c.}$ The standard (linear) stability analysis then shows that a linear modulation with the frequency Ω and the wave number κ obeys the dispersion relation

$$\Omega^2(\kappa) = p\kappa^2(p\kappa^2 - 2q|\mathcal{E}_0|^2), \tag{10}$$

which exhibits a purely growing amplitude mode if $\kappa \leq \kappa_{cr} = (q/p)^{1/2}|\mathcal{E}_0|$. The growth rate $\sigma = \mathrm{Im}(\Omega)$ attains a maximum value $\sigma_{\max} = q|\mathcal{E}_0|^2$. For $pq < 0$, on the other hand, i.e., if $\omega > \omega_{c,e}/4$, the wave is modulationally *stable*, as evident from (10).

It is known from the existing theory[15,16] that the NLSE (7) is an integrable dynamical system which admits, among others, localized solutions in the form of *envelope solitons* of the *bright* ($pq > 0$) or *dark/gray* ($pq < 0$) type. Analytical expressions for these solutions are found by inserting the trial function $\mathcal{E} = \mathcal{E}_0 \exp(i\Theta)$ in Eq. (7) and then separating real and imaginary parts in order to determine the (real) functions $\mathcal{E}_0(s,\tau)$ and $\Theta(s,\tau)$. Details on the derivation of their analytic form can be found, e.g., in Refs. 18 and 19, so only the final expressions will be given in the following. Let us retain that this *ansatz* amounts to a total electric field whose components $E_j(z,t)$ $(j=x,y)$ are essentially equal to $E_j(z,t) = 2\mathcal{E}_0 \cos(kz - \omega t + \Theta)$, where the localized field envelope amplitude $\mathcal{E}_0$ and the (small) phase shift Θ will be determined, case by case. The (copropagating) density perturbation N is then readily given by Eq. (5), viz., $N = \alpha(|\mathcal{E}|^2 - |\mathcal{E}_\infty|^2)$, where we defined for later reference the constant

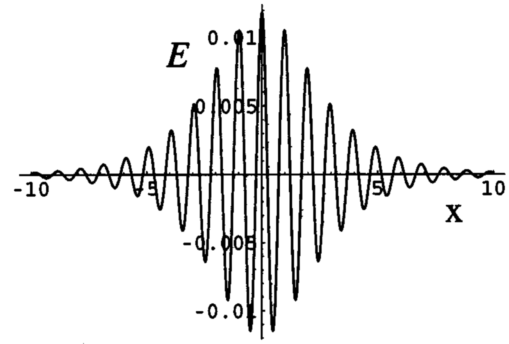
$$\alpha = \frac{\omega_{p,e}^2/\omega_{c,e}^2}{\omega/\omega_{c,e}(1 - \omega/\omega_{c,e})}. \tag{11}$$

These *bisolitons* (representing the joint propagation of $\mathcal{E}$ and N) may be of different types, which are outlined in the following.

In the modulationally unstable case ($\eta = p/q > 0$), wave collapse may result in the formation of a localized pulse-shaped slowly-varying envelope {solution of the NLSE [Eq. (7)]}, which modulates the carrier wave. This *bright*-type (*pulse*) (Fig. 1) envelope solution (with $E_\infty = 0$) is given by the expression

$$\mathcal{E}_0 = \frac{\sqrt{2\eta}}{L}\,\mathrm{sech}\left(\frac{s - v_e\tau}{L}\right), \quad \Theta = \frac{1}{2p}\left[v_e s + \left(\Omega - \frac{v_e^2}{2}\right)\tau\right], \tag{12}$$

where v_e is the envelope velocity; L and Ω represent the pulses spatial width and oscillation frequency (at rest), respectively. In our problem, the bright-type localized envelope solutions (Fig. 2) may occur and propagate in the plasma if a sufficiently long wavelength is chosen, so that the product pq is positive. We note that the pulse width L and the maximum amplitude $\hat{\mathcal{E}}_0$ satisfy $L\hat{\mathcal{E}}_0 = (2p/q)^{1/2} = \text{constant}$.



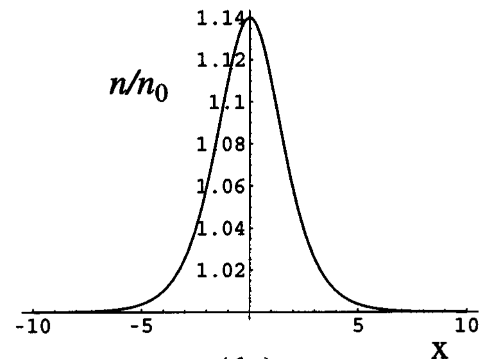


FIG. 1. Bright-type (pulse) bisoliton solution (for $pq > 0$), obtained for an indicative set of numerical values for the parameters in Eqs. (11) and (12). $\omega_{p,e}/\omega_{c,e} = 12$, $|\hat{\mathcal{E}}_0|^2/4\pi n_0 T_e = 1.4 \times 10^{-4}$ (magnetopause values), $v_e = 0.2$, and $\omega/\omega_{c,e} = 0.17$ (snapshot at $t=0$): (a) the electric field wave form $\mathcal{E}_{x,y}$ vs the (normalized) position x/λ (λ, whistler wavelength); (b) the corresponding (normalized) density $n/n_0 = 1 + N$−cf. Eq. (5).

For $\eta = p/q < 0$ (i.e., $\omega > \omega_{c,e}/4$), we have the *dark* envelope soliton (*hole*) Ref. 18 (see Fig. 3)

$$\mathcal{E}_0 = \frac{\sqrt{2|\eta|}}{L'}\left|\tanh\left(\frac{s - v_e\tau}{L'}\right)\right|,$$
$$\Theta = \frac{1}{2p}\left[v_e s + \left(2pq\hat{\mathcal{E}}_0^2 - \frac{v_e^2}{2}\right)\tau\right], \tag{13}$$

which represents a localized region of negative wave density (a *void*). As with the bright soliton (pulse) above, the excitation width L' is inversely proportional to the amplitude $\hat{\mathcal{E}}_0 = (2|\eta|)^{1/2}/L'$.

For $\eta = p/q < 0$, we also have the gray envelope soliton[18] (see Fig. 4)

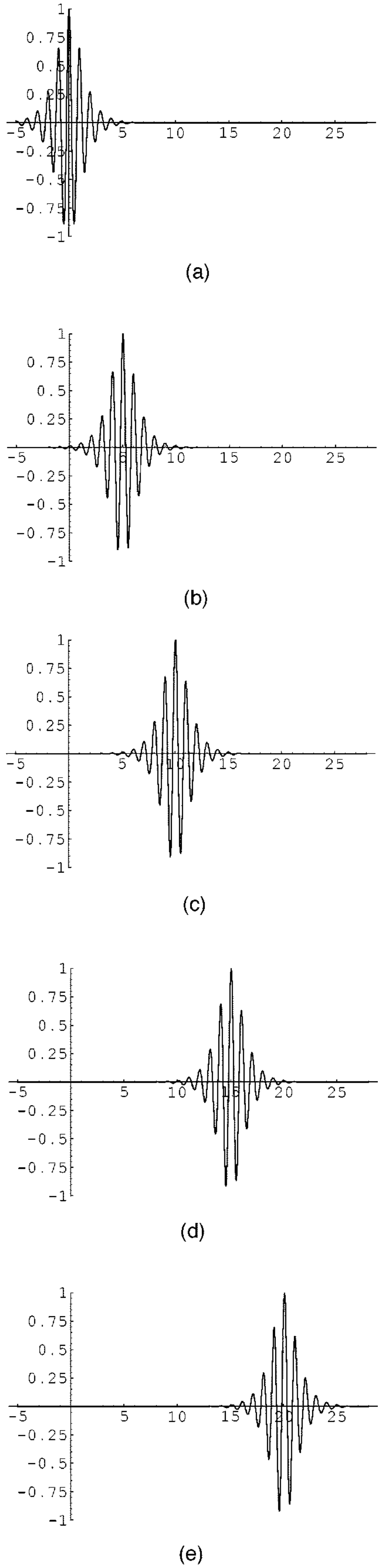


FIG. 2. Propagating bright-type whistler electric field modulated wave packet (whistleron envelope soliton excitation, for $pq>0$), as defined in Eq. (12), snapshots at successive time: $t=nt_0$ ($n=0,1,2,3,4$).

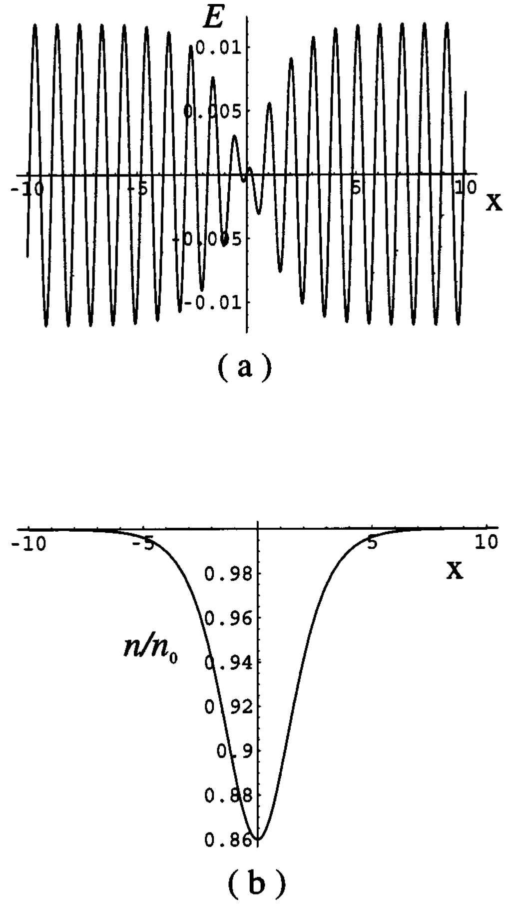


FIG. 3. Dark-type (hole) soliton solution of the NLS equation for $pq<0$; here $\omega/\omega_{c,e}=0.33$ and the remaining values are as in Fig. 2: (a) the field wave form; (b) the corresponding (normalized) density $n/n_0=1+N<1$ −cf. Eq. (5) (normalized over its asymptotic value).

$$\mathcal{E}_0 = \frac{\sqrt{2|\eta|}}{|d|L''}\{1-d^2\,\mathrm{sech}^2[(s-v_e\tau)/L'']\}^{1/2}, \tag{14}$$

where the nonlinear phase correction $\Theta=\Theta(X,T)$ now bears the complex expression (see in Ref. 18).

$$\Theta = \frac{1}{2p}\left[V_0 s - \left(\frac{1}{2}V_0^2 - 2pq\hat{\mathcal{E}}_0^2\right)\tau + \Theta_0\right] - a\sin^{-1}\frac{d\tanh\left(\frac{s-v_e\tau}{L''}\right)}{\left[1-d^2\,\mathrm{sech}^2\left(\frac{s-v_e\tau}{L''}\right)\right]^{1/2}}. \tag{15}$$

Here, Θ_0 is a constant phase; a $(=\pm 1)$ denotes the product $\mathrm{sgn}a=\mathrm{sgn}(p)\times\mathrm{sgn}(v_e-V_0)$. Again, the pulse width $L''=(|p/q|)^{1/2}/(|d|\hat{\mathcal{E}}_0)$ is inversely proportional to the amplitude $\mathcal{E}_0$, and now also depends on an independent real parameter d, which regulates the modulation depth; d is given by $d^2=1+(v_e-V_0)^2/(2pq\hat{\mathcal{E}}_0^2)\leqslant 1$. V_0 is an independent real constant which satisfies (see details in Ref. 18): $V_0-\sqrt{2|pq|\hat{\mathcal{E}}_0^2}\leqslant v_e\leqslant V_0+\sqrt{2|pq|\hat{\mathcal{E}}_0^2}$. This localized excitation represents a localized region of negative wave density (a propagating void), with finite (non zero) amplitude at $s=0$. For $|d|=1$

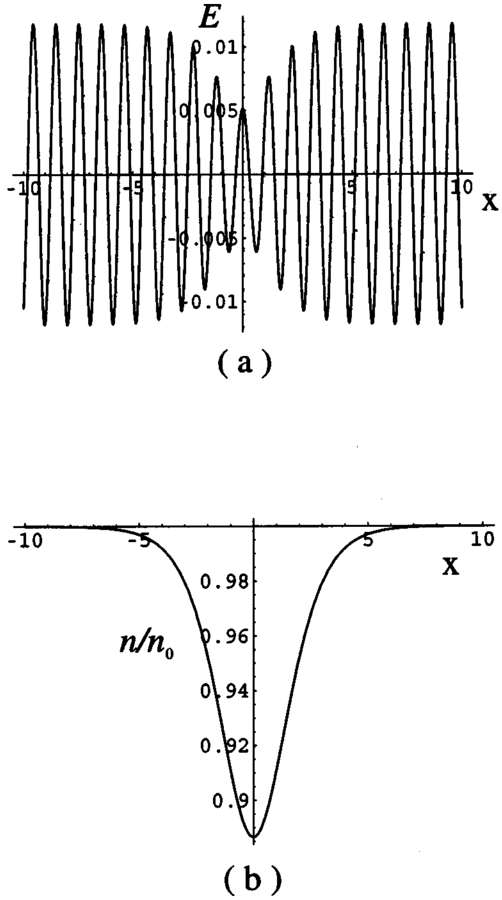


FIG. 4. Gray-type soliton solutions of the NLS equation for $pq<0$: values (and captions) identical to those in Fig. 3, in addition to $V_0=0.5$, $d=0.9$. Notice that the field amplitude never reaches zero [contrary to Fig. 3(a)].

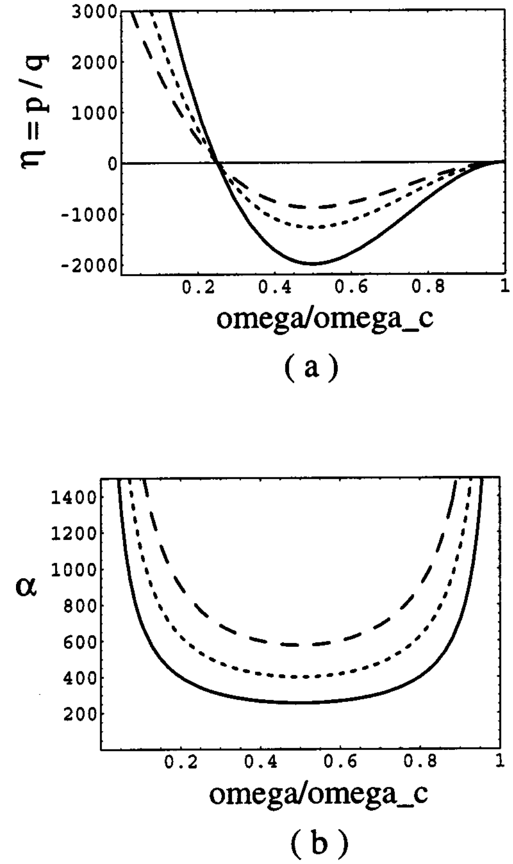


FIG. 5. The functions (a) $\eta=p/q$ and (b) α [see Eqs. (9) and (11)] are depicted vs the frequency ratio $\omega/\omega_{c,e}$, for different values of $\omega_{p,e}/\omega_{c,e}$: 8 (—), 10 (---), and 12 (---) (from bottom to top).

(thus $V_0=v_e$), one recovers the dark envelope soliton presented above, which is characterized by a vanishing field amplitude (and density perturbation) at $s=0$.

It should be stressed that the finite amplitude localized envelope excitations presented here are intrinsically different in form (and obey different physics) from the pulselike small-amplitude localized structures (solitons) typically found via the Korteweg–deVries (KdV) theory; see in Ref. 19 for a critical discussion. Also recall that those localized pulse excitations are characterized by a spatial width L and a maximum amplitude $\hat{\mathcal{E}}_0$ which satisfy $\hat{\mathcal{E}}_0 L^2=$constant, *unlike* the envelope structures above, which satisfy $\hat{\mathcal{E}}_0 L=$constant $\sim\eta$ instead [cf. the definition (9)]. This property of envelope excitations may serve as a distinguishing signature in order to identify them, e.g., in future space observation data.

Recall that the quantities η and α, in fact functions of the ratios $\omega/\omega_{c,e}$ and $\omega_{p,e}/\omega_{c,e}$ and (the former) of T_e, represent the magnitude of the field envelope and the density perturbation, respectively; cf. definitions above. In order to investigate their form numerically, we have considered a plasma-to-cyclotron frequency ratio around $\omega_{p,e}/\omega_{c,e}\approx 10$ and a temperature $T_e\approx 1$ eV, as typically reported, e.g., at the plasmapause.[3] The absolute value of η [see Fig. 5(a)] is seen to increase as the ratio $\omega_{p,e}/\omega_{c,e}\sim n_e^{1/2}B^{-1}$ decreases, suggesting that wider (i.e., spatially more extended) excitations (for a given amplitude) should be encountered for higher magnetic field B_0 or lower density n_e values (i.e., for a given value of $\omega/\omega_{c,e}$). The opposite behavior is found for α [see Fig. 5(b)], suggesting that higher B_0 (or lower n_e) should result in a smaller density perturbation $N=\alpha(|\mathcal{E}|^2-|\mathcal{E}_\infty|^2)$ for a given field amplitude excitation $\mathcal{E}$ (and fixed $\omega/\omega_{c,e}$). For any value of $\omega_{p,e}/\omega_{c,e}$, the maximum value of either η (negative) or α is attained for $\omega/\omega_{c,e}\approx 0.5$.

The localized whistler bisoliton excitations (whistlerons) presented above are depicted in Figs. 1–4. Notice in Fig. 2 the similarity to modulated wave packet forms abundantly observed in space[8] and laboratory.[7] The wave density (and energy) localization effect observed in such structures is a consequence of wave amplitude and phase modulation, which is generally due either to parametric interactions with other waves or to self-interaction of the carrier wave; this is in fact a long known phenomenon in nonlinear dynamical systems.[15,16]

IV. CONCLUSION

In conclusion, we have studied the parametric interaction between large amplitude electron-cyclotron waves (whistlers) and ponderomotively driven nonresonant density perturbations in plasmas. By adapting and then solving the associated cubic Schrödinger equation, we have shown the possibility of the occurrence of modulational instability and the existence of bright- and dark-type soliton solutions for the whistler electric field envelope. The former (bright-type solutions) represent propagating pulses modulating the electric field and associated with density humps. The latter (hole solitons) represent propagating localized voids (with non vanishing field at infinity), and are associated with localized propagating density depletion regions (voids). These results are of relevance to recent observations in space[3,4,8] and laboratory[5,7,8] plasmas, where such structures are clearly seen. A satisfactory exact theory for the formation and dynamics of these structures has always been lacking in the literature, so the present study aims in partially filling this gap.

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