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Envelope solitons associated with electromagnetic waves in a magnetized pair plasma

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The amplitude modulation of magnetic field-aligned circularly polarized electromagnetic (CPEM) waves in a magnetized pair plasma is reexamined. The nonlinear frequency shifts include the effects of the radiation pressure driven density and compressional magnetic field perturbations as well as relativistic particle mass variations. The dynamics of the modulated CPEM wave packets is governed by a nonlinear Schrödinger equation, which has attractive and repulsive interaction potentials for fast and slow CPEM waves. The modulational stability of a constant amplitude CPEM wave is studied by deriving a nonlinear dispersion from the cubic Schrödinger equation. The fast (slow) CPEM mode is modulationally unstable (stable). Possible stationary amplitude solutions of the modulated fast (slow) CPEM mode can be represented in the form of bright and dark/gray envelope electromagnetic soliton structures. Localized envelope excitations can be associated with the microstructures in pulsar magnetospheres and in laboratory pair magnetoplasmas. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1830014]

I. INTRODUCTION

Magnetized electron-positron (henceforth referred to as *e-p* or pair) plasmas, which are composed of fully ionized particles with same mass and opposite charges, exist in pulsar magnetospheres,[1–5] in the bipolar outflows (jets) in active galactic nuclear,[6] at the center of our own galaxy,[7] in the early universe,[8] and in inertial confinement fusion scheme using ultraintense lasers.[9] The probability of multiphoton production of *e-p* pairs in a plasma by electromagnetic waves has been discussed by Avetisyan *et al.*[10] Nonrelativistic pair plasmas have also been created in experiments[11] for understanding the dynamics of pairs. Recently, Helander and Ward[12] have discussed the possibility of pair production in large tokamaks due to collisions between multi-MeV runaway electrons and thermal particles.

The physics of a pair plasma is markedly different from the electron-ion (*e-i*) plasma in that many of the time and space scales, which are present in an *e-i* plasma, are simply absent in a pair plasma due to equal masses of the pairs. For example, in an unmagnetized pair plasma, the two distinct normal modes are the high-frequency electromagnetic and Langmuir waves, which interact neither linearly nor nonlinearly. In a magnetized pair plasma, besides the electrostatic upper-hybrid waves, we have the perpendicularly propagating ordinary and extraordinary modes as well as magnetic-field-aligned circularly polarized electromagnetic (CPEM) waves. Iwamoto[13] has presented an elegant description of numerous linear collective modes in a nonrelativistic pair magnetoplasma. Zank and Greaves[14] have discussed the linear properties of various electrostatic and electromagnetic modes in unmagnetized and magnetized pair plasmas, in addition to discussing the two-stream instability and nonenvelope solitary wave solutions. Magnetic-field-aligned nonlinear Alfvén waves in an ultrarelativistic pair plasma have also been investigated by Sakai and Kawata[15] and Verheest.[16] Zhao *et al.*[17] have performed three-dimensional electromagnetic particle simulations of nonlinear Alfvén waves in an electron-positron magnetoplasma.

In their classic paper, Chian and Kennel[18] considered the amplitude modulation of linearly polarized electromagnetic waves in an unmagnetized pair plasma, taking into account wave intensity induced particle mass variations. They reported the modulational instability and the formation of localized electromagnetic pulses, which may account for the pulsar microstructures.[19] The work of Chian and Kennel[18] has been extended by several authors[20–22] by including the effect of an external magnetic field, but by neglecting the CPEM wave pressure induced magnetic-field perturbations. Furthermore, we note that Weatherall[23] has considered the modulational instability of Langmuir waves as a possible mechanism for radio emission in the magnetized pair plasma of pulsars.

In this paper, we present a complete investigation of the amplitude modulation of the magnetic-field-aligned CPEM in a magnetized pair plasma by including the combined effects of the ponderomotive force driven density and magnetic-field perturbations, as well as by retaining relativistic particle mass increase in the CPEM wave fields. The dynamics of a modulated CPEM wave packet is governed by a cubic Schrödinger equation, which exhibits modulational instability and localized envelope CPEM solutions. The latter have also been discussed in contexts as diverse as solid state physics, nonlinear optics, and plasma physics.[24,25]

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The paper is organized in the following fashion. In Sec. II, we present the governing equation for the modulated CPEM wave packet and the expression for nonlinear frequency shifts associated with radiation pressure induced quasistationary plasma density and magnetic-field perturbations, as well as with that caused by relativistic particle mass increase in the CPEM waves. Section III contains the results for the modulational instability of a constant amplitude CPEM pump, and for the localized CPEM excitations in the form of bright and dark envelope solitons. Section IV presents a brief summary and possible applications.

II. THE MODEL

Let us consider a collisionless pair plasma comprising electrons and positrons with equal rest masses m_0 and opposite charges of magnitude e. The equilibrium electron/positron number density is given by n_0. The pair plasma is embedded in a homogeneous magnetic field $\mathbf{B}_0=B_0\hat{\mathbf{z}}$, where B_0 is the strength of the magnetic field and $\hat{\mathbf{z}}$ is the unit vector along the z axis of a Cartesian coordinate system. We consider the amplitude modulation of circularly polarized electromagnetic (CPEM) waves propagating along the magnetic-field direction in our pair plasma. The electric field of the CPEM waves is $\mathbf{E}=\frac{1}{2}E(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}})\exp(ikz-i\omega t)+\text{c.c.}$, where $\hat{\mathbf{x}}$ ($\hat{\mathbf{y}}$) is the unit vector along the x (y) axis, k is the wave number, and ω is the wave frequency. The linear dispersion relation for the CPEM waves is[26]

$$N^2 \equiv \frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega(\omega \mp \omega_{ce})} - \frac{\omega_{pp}^2}{\omega(\omega \mp \omega_{cp})}, \tag{1}$$

where $\omega_{pe,pp}=(4\pi ne^2/m_{e,p})^{1/2}$ and $\omega_{ce,cp}=eB/m_{e,p}c$ are the electron/positron plasma and gyrofrequencies, respectively, n is the electron/positron number density, B is the (total) compressional magnetic field, $m_{e,p}=m_0\gamma_{e,p}$ is the electron/positron mass, $\gamma_{e,p}$ are the relativistic gamma factors, and c is the speed of light in vacuum. We have carefully disentangled the electron and positron contributions because of their different relativistic mass variations. In equilibrium, however, the electron and positron plasma and gyrofrequencies are equal and the dispersion law is given by[13,14]

$$N^2 = 1 - \frac{2\omega_p^2}{\omega^2 - \omega_c^2}, \tag{2}$$

where $\omega_p=(4\pi n_0e^2/m_0)^{1/2}$ and $\omega_c=eB_0/m_0c$ are the unperturbed electron/positron plasma and gyrofrequency.

We note that Eq. (2) remains intact for both the right- and left-hand circular polarizations, and there is no Faraday rotation for these waves due to the identical index of refraction N. Solutions of Eq. (2) are

$$\omega^2 = \frac{k^2c^2 + \omega_H^2 \pm \sqrt{(k^2c^2 + \omega_H^2)^2 - 4k^2c^2\omega_c^2}}{2}, \tag{3}$$

where $\omega_H=(2\omega_p^2+\omega_c^2)^{1/2}$ is the upper-hybrid resonance frequency. The two solutions of Eq. (2) are displayed in Fig. 1, where we see the fast (frequencies higher than ω_H) and slow (frequencies below ω_c) modes.

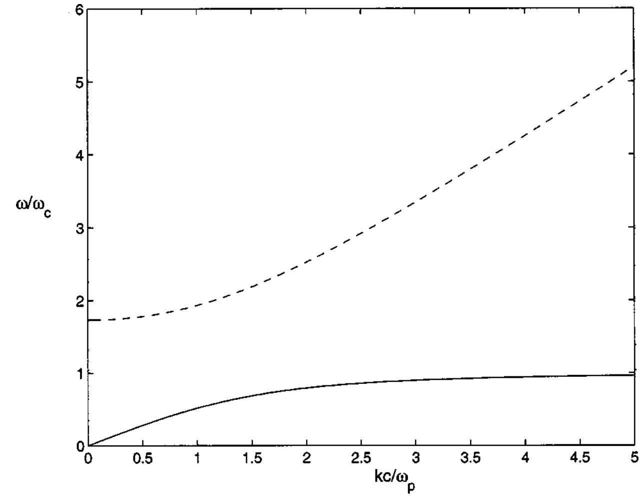

FIG. 1. Slow (full line) and fast (dotted line) modes for $\omega_p/\omega_c=1$ and $k_BT/mc^2=0.05$. These values are also used for the other plots.

We now follow the standard method of Karpman and Washimi[27] and Nishikawa and Liu[28] to derive an equation for the amplitude modulated CPEM waves in the presence of nonlinear frequency shifts caused by the radiation pressure driven density and magnetic-field perturbations as well as that associated with relativistic particle mass increase in the CPEM waves. Thus, we shall account for the coupling of the CPEM waves with the plasma slow motion and consider only a weakly relativistic particle mass variation case. Due to weakly nonlinear effect under consideration, the dispersion relation shall depend on the wave amplitude, while the wave group velocity and the wave group dispersion are supposed to remain unchanged. Within the Wentzel–Kramers–Brillouin (WKB) approximation, modulation may be expressed in terms of a slow space-time variation of the amplitude around some average value of the CPEM wave electric field.[28] We now consider Eq. (1) to be a nonlinear dispersion relation $\omega=\omega(k,n_0+n_1,B_0+B_1,m_0\gamma_e,m_0\gamma_p)$, where n_1 and B_1 are the radiation pressure driven density and compressional (along the z axis) magnetic-field perturbations, associated with the plasma slow motion, respectively, $\gamma_e\approx 1+|\mathbf{v}_e|^2/2c^2$ and $\gamma_p\approx 1+|\mathbf{v}_p|^2/2c^2$ are the relativistic gamma factors, and $\mathbf{v}_e$ ($\mathbf{v}_p$) is the electron (positron) quiver velocity in the CPEM waves. For our purposes, we have $\mathbf{v}_e=-i[e/m_0(\omega\mp\omega_c)]E(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}})$ and $\mathbf{v}_p=i[e/m_0(\omega\pm\omega_c)]E(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}})$. We assume that the radiation pressure density perturbations are quasineutral and quasistationary. By Taylor expanding the dispersion relation, we then have

$$\omega \simeq \omega_0 + \frac{\partial\omega}{\partial k}(k-k_0) + \frac{1}{2}\frac{\partial^2\omega}{\partial k^2}(k-k_0)^2 + \frac{\partial\omega}{\partial n_1}n_1 + \frac{\partial\omega}{\partial B_1}B_1 + \frac{\partial\omega}{\partial v_e^2}v_e^2 + \frac{\partial\omega}{\partial v_p^2}v_p^2, \tag{4}$$

which can be written in the form

$$\omega \simeq \omega_0 + v_g(k-k_0) + P(k-k_0)^2 + \Delta. \tag{5}$$

By replacing $i(k-k_0)\to\partial/\partial z$ and $-i(\omega-\omega_0)\to\partial/\partial t$, we readily obtain a nonlinear Schrödinger equation[27]

$$i\left(\frac{\partial E}{\partial t}+v_g\frac{\partial E}{\partial z}\right)+P\frac{\partial^2 E}{\partial z^2}-\Delta E=0. \qquad (6)$$

Here, lowest order group velocity and group velocity dispersion of the carrier CPEM wave are, respectively,

$$v_g=\left(\frac{\partial\omega}{\partial k}\right)_{k=k_0}=\frac{kc^2}{\omega\left[1+\dfrac{2\omega_c^2\omega_p^2}{(\omega^2-\omega_c^2)^2}\right]}>0 \qquad (7)$$

and

$$P=\frac{1}{2}\left(\frac{\partial^2\omega}{\partial k^2}\right)_{k=k_0}=\frac{v_g}{2k}\left\{1+\frac{v_g^2}{c^2}\left[\frac{2\omega_c^2\omega_p^2(3\omega^2+\omega_c^2)}{(\omega^2-\omega_c^2)^3}-1\right]\right\}. \qquad (8)$$

It is straightforward to check numerically that $P>0$ ($P<0$) for the fast (slow) CPEM mode.

The nonlinear frequency shift is

$$\Delta=\frac{\partial\omega}{\partial N^2}\left[\left(\frac{\partial N^2}{\partial n_1}\right)_{n=n_0}n_1+\left(\frac{\partial N^2}{\partial B_1}\right)_{B=B_0}B_1+\left(\frac{\partial N^2}{\partial v_e^2}\right)_{v_e^2=0}v_e^2+\left(\frac{\partial N^2}{\partial v_p^2}\right)_{v_p^2=0}v_p^2\right]. \qquad (9)$$

A simple calculation gives

$$\frac{\partial\omega}{\partial N^2}=-\frac{\partial\omega}{\partial k}\left(\frac{\partial N^2}{\partial k}\right)^{-1}=-\frac{v_g\omega^2}{2kc^2}. \qquad (10)$$

Next, we carry out the expansion of the dispersion relation (1) by letting $\omega_{pe,pp}^2=4\pi(n_0+n_1)e^2(1-|\mathbf{v}_{e,p}|^2/2c^2)/m_0$ and $\omega_{ce,cp}=e(B_0+B_1)(1-|\mathbf{v}_{e,p}|^2/2c^2)/m_0c$. To leading order, we have

$$\left(\frac{\partial N^2}{\partial n_1}\right)_{n=n_0}=-\frac{2\omega_p^2}{\omega^2-\omega_c^2}\frac{1}{n_0}, \qquad (11)$$

$$\left(\frac{\partial N^2}{\partial B_1}\right)_{B=B_0}=-\frac{4\omega_p^2\omega_c^2}{(\omega^2-\omega_c^2)^2}\frac{1}{B_0}, \qquad (12)$$

$$\left(\frac{\partial N^2}{\partial v_e^2}\right)_{v_e^2=0}=\frac{\omega_p^2}{2c^2(\omega\mp\omega_c)^2}, \qquad (13)$$

$$\left(\frac{\partial N^2}{\partial v_p^2}\right)_{v_p^2=0}=\frac{\omega_p^2}{2c^2(\omega\pm\omega_c)^2}. \qquad (14)$$

We could now write the frequency shift in the form[23]

$$\Delta=\frac{v_g}{kc^2}\frac{\omega^2\omega_p^2}{\omega^2-\omega_c^2}\left(\frac{n_1}{n_0}+\frac{2\omega_c^2}{\omega^2-\omega_c^2}\frac{B_1}{B_0}\right)+\Delta_r, \qquad (15)$$

where Δ_r is the relativistic frequency shift.

In order to calculate the density perturbations, we write the electron and positron equations of (slow) motion as

$$0=en_0\frac{\partial\varphi}{\partial z}-k_BT_e\frac{\partial n_1}{\partial z}-\frac{\omega_p^2}{4\pi\omega(\omega-\omega_c)}\frac{\partial|E|^2}{\partial z} \qquad (16)$$

and

$$0=-en_0\frac{\partial\varphi}{\partial z}-k_BT_p\frac{\partial n_1}{\partial z}-\frac{\omega_p^2}{4\pi\omega(\omega+\omega_c)}\frac{\partial|E|^2}{\partial z}, \qquad (17)$$

where φ is the ambipolar potential associated with the plasma slow motion, k_B is the Boltzmann constant, $T_e(T_p)$ is the electron (positron) temperature, and the last terms in Eqs. (16) and (17) represent the ponderomotive force[29] of the CPEM waves. Adding Eqs. (16) and (17) and integrating once, we obtain

$$\frac{n_1}{n_0}=-\frac{2\omega_p^2}{\omega^2-\omega_c^2}\frac{|E|^2-|E_0|^2}{4\pi n_0k_BT}, \qquad (18)$$

where $T=T_e+T_p$, and $|E_0|$ is a constant value of the CPEM wave envelope at $|z|=\pm\infty$.

The magnetic-field perturbation can be calculated by splitting the magnetization formula $B=H+4\pi M$ in its equilibrium $B_0=H_0+4\pi M_0$ and perturbation $B_1=4\pi M_1$ parts. The magnetization induced by the em waves is given by[30]

$$M=\frac{1}{4\pi}\frac{\partial N^2}{\partial B_0}|E|^2. \qquad (19)$$

A simple calculation for our purposes gives[22]

$$\frac{B_1}{B_0}=-\frac{4\omega_p^4}{(\omega^2-\omega_c^2)^2}\frac{|E|^2-|E_0|^2}{4\pi m_0c^2}. \qquad (20)$$

The quiver velocities are given by

$$|v_e|^2=\frac{2e^2(|E|^2-|E_0|^2)}{m_0^2(\omega\mp\omega_c)^2}, \qquad (21)$$

$$|v_p|^2=\frac{2e^2(|E|^2-|E_0|^2)}{m_0^2(\omega\pm\omega_c)^2}. \qquad (22)$$

Hence, the relativistic frequency shift is found to be[22]

$$\Delta_r=-\frac{v_g}{kc^2}\frac{\omega^2\omega_p^2[4\omega^2\omega_c^2+(\omega^2+\omega_c^2)^2]}{(\omega^2-\omega_c^2)^4}\frac{e^2(|E|^2-|E_0|^2)}{m_0^2c^2}. \qquad (23)$$

Finally, combining Eqs. (15), (18), (20), and (23), the nonlinear frequency shift can be written as $\Delta=-Q(|E|^2-|E_0|^2)$, where the coefficient of the nonlinearity can be expressed as

$$Q=\frac{v_g}{kc^2}\frac{\omega^2\omega_p^4}{(\omega^2-\omega_c^2)^2 4\pi n_0k_BT}\times\left[2+\frac{4\omega_c^2(\omega^2+2\omega_p^2)+(\omega^2+\omega_c^2)^2}{(\omega^2-\omega_c^2)^2}\frac{k_BT}{m_0c^2}\right]>0. \qquad (24)$$

The electric field envelope of the CPEM waves then obeys a nonlinear Schrödinger equation

$$i\frac{\partial E}{\partial t}+P\frac{\partial^2 E}{\partial\zeta^2}+Q(|E|^2-|E_0|^2)E=0, \qquad (25)$$

where we have introduced a Galilean transformation $z\rightarrow\zeta=z-v_gt$ to a frame moving at the group velocity. The system of Eqs. (18), (20), and (25) provides a self-consistent de-

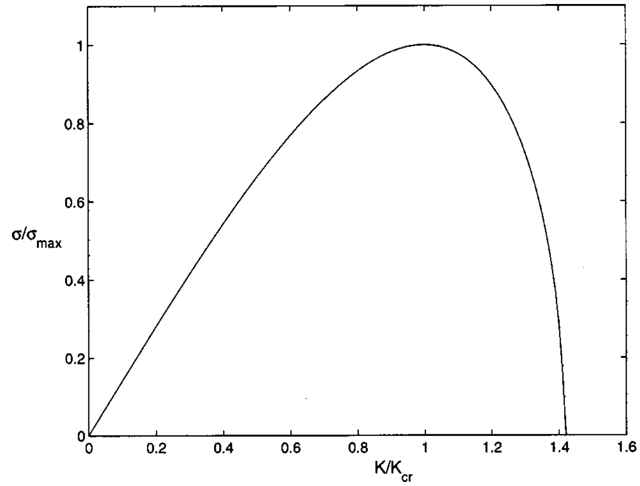

FIG. 2. The growth rate of the slow mode for $kc/\omega_p = 0.2$, $\omega/\omega_c = 1.7398$.

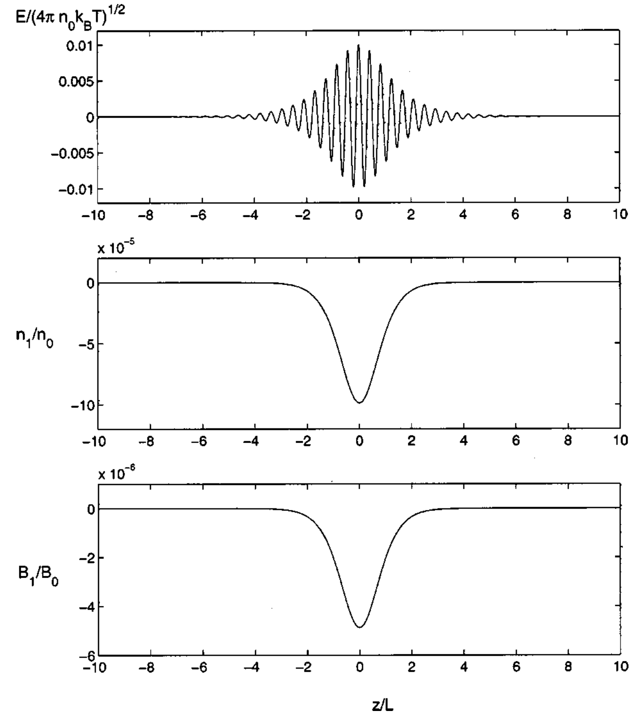

FIG. 3. The electric field, density and magnetic-field profiles of the *bright* soliton at $t=0$. We are considering the fast mode at $kc/\omega_p = 0.2$, $\omega/\omega_c = 1.7398$ and have chosen $|E_m|/(4\pi n_0 k_B T)^{1/2} = 0.01$, $v_e = 0$, $\zeta_0 = 0$ and $\theta_0 = 0$. When choosing these parameters, care has been taken to meet the requirements of the weak nonlinearity, the slowly varying amplitude and the group and envelope velocities being much smaller than the speed of light.

scription of the magnetic-field-aligned CPEM waves interacting with the plasma slow motion that supports nonresonant density and magnetic-field perturbations. The term involving $|E_0|^2$, which represents the equilibrium electric field amplitude to be determined for a given anticipated solution, could be eliminated from Eq. (25)—but not from Eqs. (18), (20), and (23), by introducing a phase shift $E \to E\exp(iQ|E_0|^2 t)$, but we prefer not to do this in order not to obscure the physical interpretation.

III. MODULATIONAL INSTABILITY AND SOLITON SOLUTIONS

In order to investigate the modulational instability of a constant amplitude CPEM wave in a pair magnetoplasma, we follow Hasegawa[25] and introduce a polar representation, i.e., we write $E=\rho\exp(i\theta)$, where $\rho(\zeta,t)$ and $\theta(\zeta,t)$ are real functions. Note that this means that $\rho=|E|$ with an equilibrium value $\rho_0=|E_0|$. Equation (25) has the obvious solution $\rho=\rho_0$, $\theta=0$. By assuming $\rho=\rho_0+\rho_1\exp(iK\zeta-i\Omega t)$ and $\theta=\sigma_1\exp(iK\zeta-i\Omega t)$, we readily obtain from Eq. (25) the nonlinear dispersion relation

$$\Omega^2 = PK^2(PK^2 - 2Q\rho_0^2) = (PK^2 - Q\rho_0^2)^2 - Q^2\rho_0^4. \qquad (26)$$

We see from Eq. (21) that if $\eta \equiv P/Q > 0$, Ω^2 becomes negative for values of K below $K_{cr} = (2/\eta)^{1/2}|E_0|$, so that there is a purely growing mode and the CPEM wave is modulationally unstable. The growth rate $\sigma = \mathrm{Im}(\Omega)$ attains a maximum $\sigma_{\max} = Q|E_0|^2$ for $K=(1/\eta)^{1/2}|E_0|$. The variation of the growth rate is displayed in Fig. 2. On the other hand, for $\eta<0$, the CPEM wave is modulationally stable.

A. The fast mode

As we saw above, the fast mode has $\eta>0$, and is thus modulationally unstable. Possible final states of the latter can lead to the formation of a bright soliton,[31] i.e., a localized pulselike envelope modulating the carrier wave. It is of the form $E=\rho\exp(i\theta)$, where

$$\rho = \frac{\sqrt{2\eta}}{L}\,\mathrm{sech}\left(\frac{\zeta-\zeta_0-Ut}{L}\right) \qquad (27)$$

and

$$\theta = \frac{1}{2P}\left[u\zeta + \left(\frac{\sqrt{2}P}{L} - \frac{U^2}{2}\right)t\right] + \theta_0. \qquad (28)$$

Here U is the envelope speed, L and ζ_0 are the pulses spatial width and position at $t=0$, and θ_0 is an arbitrary phase. The equilibrium amplitude is $\rho_0=0$. The maximum amplitude ρ_M is inversely proportional to the width L, i.e., $\rho_M L = (2\eta)^{1/2}$. Note that when going back to the original coordinates, the envelope is moving at the speed v_g+U. Let us also emphasize that the total phase of the electric field is $kz-\omega t+\theta$. Furthermore, we can calculate the density and magnetic-field profiles with the help of the Eqs. (18) and (20). It is found that these bright solitons correspond to a reduction of both the density and magnetic field, as seen in Fig. 3. The time evolution of the bright soliton is depicted in Fig. 4.

B. The slow mode

The slow mode is modulationally stable since $\eta<0$. Now there are dark and gray envelope solitons[31] which represent a localized region of reduced electric field density. The dark has

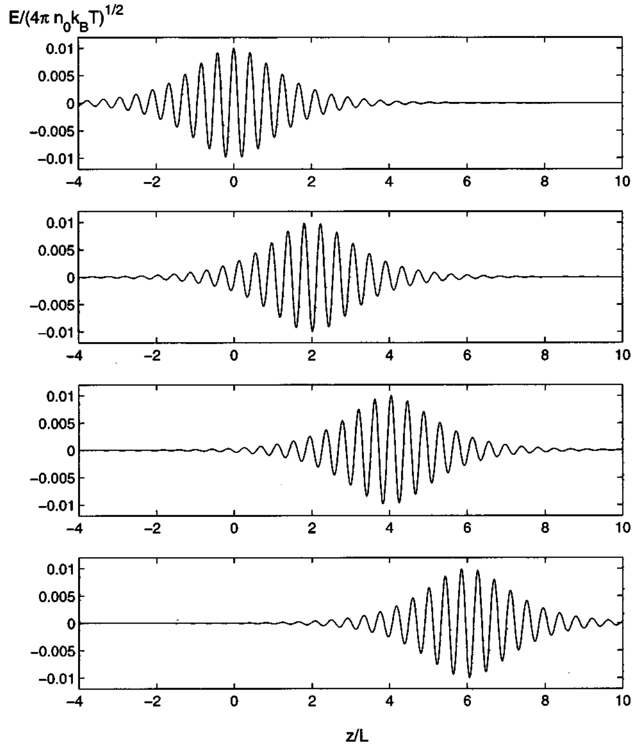

FIG. 4. The electric field profiles of the *bright* soliton at successive times $tL/v_g = 0, 2, 4, 6$. The parameters are the same as in the previous figure.

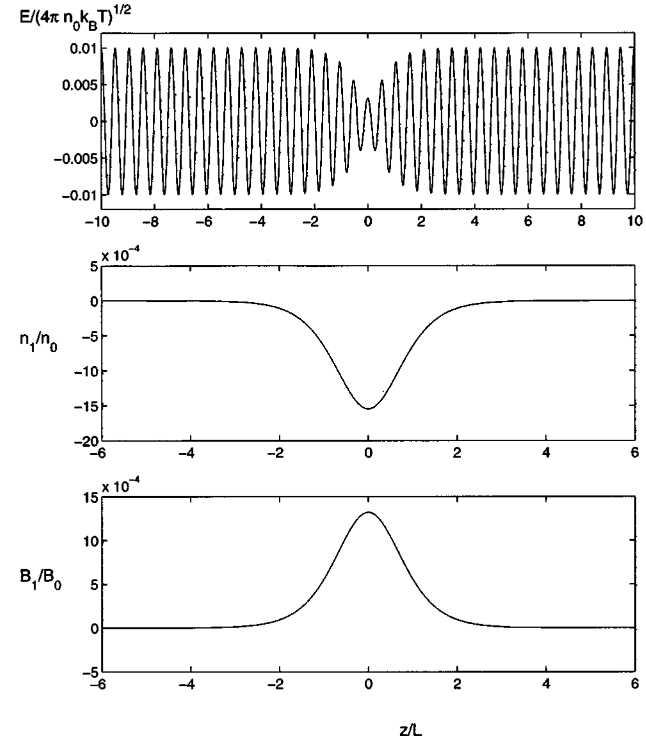

FIG. 5. The electric field, density, and magnetic-field profiles of the gray soliton at $t=0$. We are considering the slow mode at $kc/\omega_p=4$, $\omega/\omega_c=0$, 9398, and have chosen $|E_0|/(4\pi n_0 k_B T)^{1/2}=0.01$, $v_e=0$, $d=0.95$, $s=+1$, $\zeta_0=0$, and $\theta_0=0$.

$$\rho = \frac{\sqrt{2|\eta|}}{L}\tanh\left(\frac{\zeta-\zeta_0-Ut}{L}\right) \tag{29}$$

and

$$\theta = \frac{1}{2P}\left(U\zeta - \frac{U^2}{2}t\right) + \theta_0, \tag{30}$$

while the gray has

$$\rho = \frac{\sqrt{2|\eta|}}{Ld}\sqrt{1 - d^2\,\mathrm{sech}^2\left(\frac{\zeta-\zeta_0-Ut}{L}\right)}, \tag{31}$$

and

$$\theta = \frac{1}{2P}\left(V_0\zeta - \frac{V_0^2}{2}t\right) + s\arcsin\frac{d\tanh\left(\frac{\zeta-Ut}{L}\right)}{\sqrt{1-d^2\,\mathrm{sech}^2\left(\frac{\zeta-Ut}{L}\right)}} + \theta_0. \tag{32}$$

Here the parameter d, lying in the range $0<d\leqslant 1$, regulates the modulation depth, $s=\pm 1$, and V_0 is given by the formula $V_0=U+s(2P/dL)\sqrt{1-d^2}$. Note that for $d=1$, one recovers the dark soliton. The (finite) equilibrium amplitude ρ_0 is now inversely proportional to both the width L and the parameter d, i.e., $\rho_0 dL=(2|\eta|)^{1/2}$. The minimum amplitude ρ_m is given by $\rho_m=\rho_0(1-d^2)^{1/2}$, which is zero in the dark case. These dark and gray solitons correspond to a decrease in the density and an increase of the magnetic field, as seen in Fig. 5.

IV. CONCLUSIONS

We have reexamined the amplitude modulation of magnetic-field-aligned CPEM waves in a magnetized pair plasma, taking into account relativistic particle mass increase in the wave field, as well as the modification of the equilibrium density and magnetic field profiles by the radiation pressure. The dynamics of the modulated CPEM wave packet is governed by a nonlinear Schrödinger equation. The latter is analyzed to obtain the nonlinear dispersion relation for the modulational instability of a constant amplitude CPEM pump. It is found that the fast (slow) wave is modulationally unstable (stable). Possible stationary solutions of the nonlinear Schrödinger equation can be represented in the form of a bright (dark/gray) envelope soliton for the fast (slow) mode. The present results suggest that the nonlinear coupling between the CPEM waves with the background plasma provides the possibility of localized electromagnetic pulses which can propagate over long distances. In fact, localized slow mode excitations can be associated with microstructures in pulsar magnetospheres where the electron gyrofrequency is much larger than the pulsar frequency. On the other hand, the nonlinear effects associated with the fast CPEM mode may be relevant to some laboratory experiments devoted to fundamental studies of collective interactions in magnetized pair plasmas, as well as to laser-produced pair plasmas where the electron plasma and electron gyrofrequencies could be similar. Future observations may lend support to our prediction of nonlinear envelope excitations in magnetized pair plasmas found in astrophysical settings and in low-temperature laboratory environments.

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