Eur. Phys. J. D **28**, 109–117 (2004)
DOI: 10.1140/epjd/e2003-00292-4

THE EUROPEAN PHYSICAL JOURNAL D

Finite ion temperature effects on oblique modulational stability and envelope excitations of dust-ion acoustic waves

I. Kourakis[a] and P.K. Shukla

Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr–Universität Bochum, 44780 Bochum, Germany

Received 2 September 2003
Published online 21 October 2003 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2004

Abstract. Theoretical and numerical investigations are carried out for the amplitude modulation of dust-ion acoustic waves (DIAW) propagating in an unmagnetized weakly coupled collisionless fully ionized plasma consisting of isothermal electrons, warm ions and charged dust grains. Modulation oblique (by an angle θ) to the carrier wave propagation direction is considered. The stability analysis, based on a nonlinear Schrödinger-type equation (NLSE), exhibits a sensitivity of the instability region to the modulation angle θ, the dust concentration and the ion temperature. It is found that the ion temperature may strongly modify the wave's stability profile, in qualitative agreement with previous results, obtained for an electron-ion plasma. The effect of the ion temperature on the formation of DIAW envelope excitations (envelope solitons) is also discussed.

PACS. 52.27.Lw Dusty or complex plasmas; plasma crystals – 52.35.Fp Electrostatic waves and oscillations (e.g., ion-acoustic waves) – 52.35.Mw Nonlinear phenomena: waves, wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.) – 52.35.Sb Solitons; BGK modes

1 Introduction

In the recent few years, dusty plasmas (DP) have attracted increasing attention due to a realm of new phenomena associated to them and the exciting novel physics involved in their description [1]. Of particular interest was the theoretical prediction [2,3] and subsequent experimental confirmation [4–6] of the existence of new DP oscillatory modes, namely the dust-acoustic wave (DAW) and the dust-ion acoustic wave (DIAW) [1,7]. The latter, which is the object of this study, relies on a physical mechanism quite analogous to that of the ion acoustic wave (IAW): inertialess thermalized electrons provide the restoring force, while massive ions provide the inertia. The DIAW is characterized by a phase velocity which is much smaller (larger) than the ion (electron) thermal speed, and a frequency which is higher than the dust plasma frequency $\omega_{p,d}$; therefore, on the timescale of relevance, stationary dust does not participate in the wave dynamics. In fact, the DIAW phase velocity is higher than that of IA waves, because of the electron density depletion in the background plasma when dust grains are negatively charged; remarkably, this fact results in suppression of the Landau damping mechanism [1], which is known to prevail over the IAW propagation in an electron-ion plasma [8,9].

Wave propagation in a nonlinear medium like plasma is generically subject to amplitude modulation due to the carrier wave self-interaction, related to the harmonic generation. The standard reductive perturbation technique [10,11] used to study this mechanism, leads to a nonlinear Schrödinger-type equation (NLSE), which describes the evolution of the carrier wave envelope. Modulated waves may develop a Benjamin-Feir-type (modulational) instability (MI), i.e. envelope collapses when subjected to external perturbations, a mechanism which often favors energy localization via the formation of envelope localized structures (envelope solitons), as is known from a variety of physical contexts [12–15]. These long-lived localized excitations are sustained by a mutual compensation of dispersion and nonlinearity and can propagate in the medium over long distances, remarkably surviving impacts with each other.

Plasma electrostatic modes have been widely studied in this respect [10,11,16–24]. In a rather general fashion [25], these studies have revealed the existence of a (carrier wavenumber) instability threshold, which is found to change once oblique modulation [19–21] or temperature effects [22–24] are taken into account. As far as DP electrostatic modes are concerned, studies of both DAW [26,27] and DIAW [26,29,30] have been carried out,

[a] *On leave from*: U.L.B., Université Libre de Bruxelles, Faculté des Sciences Apliquées, C.P. 165/81 Physique Générale, avenue F.D. Roosevelt 49, 1050 Brussels, Belgium.
e-mail: ioannis@tp4.rub.de

followed by similar investigations of (strongly-coupled) DP lattice modes [31,32]. However, all known studies of DIAW modulation are limited to a cold-ion description, where the ion pressure effect is omitted, for simplicity. This fact may favor analytical tractability, but overlooks the interesting ion temperature effects known from the IAW case (in an electron-ion plasma), since experiments on the IAW instability [35] and on the IA/DIA solitary wave formation [36–38] have revealed important sensitivity to the ion temperature.

In this paper, we aim in generalizing previous results [30] by studying the modulational instability of dust-ion acoustic waves propagating in an unmagnetized fully ionized dusty plasma consisting of *warm* ions, Boltzmann distributed electrons and massive charged dust grains, whose dimensions and charge are assumed to be constant, for simplicity. Amplitude modulation is allowed to take place in an oblique direction, at an angle θ with respect to the carrier wave propagation direction. Our aim is multifold. Once the conditions for instability to occur are established, we aim in examining their dependence on physical parameters as the angle θ, the dust number density n_d and (focusing on) the ion temperature T_i. Finally, the possibility of the formation of envelope solitary waves will be addressed.

2 The formalism

We consider a three component collisionless unmagnetized dusty plasma consisting of electrons (mass m, charge e), ions (mass m_i, charge $q_i = +Z_i e$) and heavy dust particulates (mass m_d, charge $q_d = sZ_d e$), henceforth denoted by e, i, d respectively. The inertial (heavy) dust particles are assumed to be practically immobile ($n_d \approx n_{d,0}$) since the DIA wave is characterized by timescales much shorter than the dust plasma period ($\sim \omega_{p,d}^{-1}$) [1]. Dust mass and charge will be taken to be constant, for simplicity. Note that both negative and positive dust charge cases are considered, distinguished by the charge sign $s = \mathrm{sgn}\, q_d = \pm 1$.

2.1 Model equations

The system of (reduced) moment-Poisson evolution equations for the ions reads

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0,$$
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \phi - \frac{\sigma}{n} \nabla p,$$
$$\frac{\partial p}{dt} + \mathbf{u} \cdot \nabla p = -\gamma p \nabla \cdot \mathbf{u},$$

where all quantities are dimensionless: n, $\mathbf{u}$ and p, respectively, denote the (normalized) dust density $n_d/n_{i,0}$, mean velocity $\mathbf{u}_d/c_s = [m_i/(k_B T_e)]^{1/2}\mathbf{u}_d$ and pressure $p_i/(n_{i,0} k_B T_i)$; $\gamma = (f+2)/f$ is the ratio of specific heats (f is the number of degrees of freedom) e.g. $\gamma = 3$ in the adiabatic one-dimensional (1d) case and $\gamma = 5/3$ in three dimensions; space and time are, respectively, scaled over the electron Debye radius $\lambda_{D,e} = (k_B T_e/4\pi n_{e,0} e^2)^{1/2}$, and $t_0 = \lambda_{D,e}/c_s \equiv \omega_{p,e}^{-1}(m_i/m_e)$, where $\omega_{p,e}$ denotes the electron plasma frequency $\omega_{p,e} = (4\pi n_{e,0} e^2/m_e)^{1/2}$. The (reduced) electric potential $\phi = Z_i e\Phi/(k_B T_e)$ obeys Poisson's equation: $\nabla^2 \Phi = -4\pi \sum q_\alpha n_\alpha$, which here takes the form

$$\nabla^2 \phi = \phi + \alpha\phi^2 + \alpha'\phi^3 - \beta(n-1), \tag{1}$$

by linearizing around a Boltzmann state assumed for electrons, i.e. $n_e \approx n_{e,0} e^{e\Phi/k_B T_e}$ (T_α is the temperature of species $\alpha = e, i$; k_B is the Boltzmann constant). The dimensionless parameters are conveniently expressed in terms of typical dust parameters, i.e. either the ratio $\mu = n_{e,0}/(Z_i n_{i,0})$ or $\delta = (Z_d n_{d,0})/(Z_i n_{i,0})$. One has: $\alpha = 1/(2Z_i)$, $\alpha' = 2\alpha^2/3 = 1/(6Z_i^2)$ and $\beta = Z_i^2 n_{i,0}/n_{e,0} \equiv 1/(2\alpha\mu) = Z_i/\mu$. Since overall neutrality is assumed at equilibrium: $n_{e,0} - Z_i n_{i,0} - sZ_d n_{d,0} = 0$, one has $\mu = 1 + s\delta$, so that $0 \le \mu < 1$ ($\mu > 1$) corresponds to negative (positive) dust charge; obviously, $\mu = 1$ (as $\delta = 0$) in the absence of dust (previous results for the ion-acoustic wave in an electron-ion plasma are recovered in this limit) [33]. Finally, σ denotes the temperature ratio T_i/T_e; taking $\sigma \to 0$ one recovers the "cold ion" DIAW model used previously [26,30].

2.2 Multiple scales perturbation method

Let $\mathbf{S}$ be the state (column) vector $(n, \mathbf{u}, p, \phi)^T$, describing the system's state at a given position $\mathbf{r}$ and instant t. We shall consider small deviations from the equilibrium state $\mathbf{S}^{(0)} = (1, \mathbf{0}, 1, 0)^T$ by taking $\mathbf{S} = \mathbf{S}^{(0)} + \epsilon \mathbf{S}^{(1)} + \epsilon^2 \mathbf{S}^{(2)} + ... = \mathbf{S}^{(0)} + \sum_{n=1}^{\infty} \epsilon^n \mathbf{S}^{(n)}$, where $\epsilon \ll 1$ is a smallness parameter. Following the standard multiple scale (reductive perturbation) technique [10], we shall consider the stretched (slow) space and time variables $\zeta = \epsilon(x - Vt)$, $\tau = \epsilon^2 t$ ($V \in \Re$). The perturbed states are assumed to depend on the fast scales via the carrier phase $\theta_1 = \mathbf{k} \cdot \mathbf{r} - \omega t$, while the slow scales enter the argument of the jth element's lth harmonic amplitude $S_{j,l}^{(n)}$, allowed to vary along x, viz. $Sj^{(n)} = \sum_{l=-\infty}^{\infty} S_{j,l}^{(n)}(\zeta, \tau) e^{il(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ (where $S_{j,-l}^{(n)} = S_{j,l}^{(n)^*}$). The amplitude modulation (along the x-axis) is thus allowed to take place in an oblique direction, with respect to the (arbitrary) propagation direction; accordingly, the wavenumber $\mathbf{k}$ is $(k_x, k_y) = (k\cos\theta, k\sin\theta)$. Treating the derivative operators as

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \epsilon V \frac{\partial}{\partial \zeta} + \epsilon^2 \frac{\partial}{\partial \tau},$$
$$\nabla \to \nabla + \epsilon \hat{x} \frac{\partial}{\partial \zeta},$$
$$\nabla^2 \to \nabla^2 + 2\epsilon \frac{\partial^2}{\partial x \partial \zeta} + \epsilon^2 \frac{\partial^2}{\partial \zeta^2},$$

and substituting into the system of evolution equations, one obtains an infinite series in both (perturbation order)

$$Q_0 = +\frac{1}{2\omega}\frac{1}{\beta^2}\frac{1}{(1+k^2)^2}\frac{1}{\beta+\gamma\sigma-v_g^2}\Bigg\{\beta k^2\Big[\beta\big[3+6k^2+4k^4+k^6-2\alpha\beta\big(2k^2+3-2\alpha v_g^2\big)\big]$$
$$+\gamma\sigma\big[(\gamma+1)(1+k^2)^3-2\alpha\beta\big(2\alpha\beta+\gamma(1+k^2)^2\big)\big]+\big[\beta(2+4k^2+3k^4+k^6-2\alpha\beta)+2\gamma\sigma(1+k^2)^2(1+k^2-\alpha\beta)\big]\cos 2\theta\Big]$$
$$+2(1+k^2)^4(\beta+\gamma\sigma)\omega^2\cos^2\theta+k(1+k^2)\Big[\beta k^2+\omega^2(1+k^2)\Big]\frac{v_g}{\omega}\Big[\beta(1+k^2-2\alpha\beta)+\gamma(\gamma-1)\sigma(1+k^2)^2\Big]\cos\theta\Bigg\}, \quad (6)$$

$$Q_1 = \frac{3\alpha'\beta}{2\omega}\frac{k^2}{(1+k^2)^2}, \quad (7)$$

$$Q_2 = -\frac{1}{12\beta^3}\frac{1}{\omega}\frac{1}{k^2(1+k^2)^2}\Bigg\{2\beta k^2\Big[-5\alpha\beta^2(1+k^2)^2+2\alpha^2\beta^3+2\gamma^2\sigma(1+k^2)^4(1+4k^2)$$
$$+\beta(1+k^2)^3(3+9k^2-2\alpha\gamma^2\sigma)\Big]+(1+k^2)^3\omega^2\Big[\beta(3+9k^2+6k^4-2\alpha\beta)+2\gamma^2\sigma(1+k^2)^2(1+4k^2)\Big]\Bigg\} \quad (8)$$

ϵ^n and (phase harmonic) l. The standard perturbation procedure now consists in solving in successive orders $\sim\epsilon^n$ and substituting in subsequent orders. The calculation, particularly lengthy yet perfectly straightforward, follows exactly the method we have reported elsewhere [28,30], so only the essential steps are provided here. The ($n = 2$, $l = 1$) equations determine the first harmonics of the perturbation

$$n_1^{(1)} = \frac{1+k^2}{\beta}\phi_1^{(1)} = \frac{1}{\gamma}p_1^{(1)} = \frac{1}{\omega}\mathbf{k}\cdot\mathbf{u}_1^{(1)}$$
$$= \frac{k}{\omega\cos\theta}u_{1,x}^{(1)} = \frac{k}{\omega\sin\theta}u_{1,y}^{(1)} \quad (2)$$

and provides the compatibility condition: $\omega^2 = \beta k^2/(k^2+1)+\gamma\sigma k^2$, which exactly recovers, by restoring dimensions, the known DIAW dispersion relation [1,7]:

$$\omega^2 = \frac{c_D^2 k^2}{1+k^2{\lambda_D}^2}+\gamma v_{th}^2 k^2. \quad (3)$$

Proceeding in the same manner, we obtain the second order quantities, namely the amplitudes of the second harmonics $\mathbf{S}_2^{(2)}$ and constant ("direct current") terms $\mathbf{S}_0^{(2)}$, as well as a finite contribution $\mathbf{S}_1^{(2)}$ to the first harmonics; the lengthy expressions are omitted here for brevity. The ($n = 2$, $l = 1$) equations provide the compatibility condition:

$$V = v_g(k) = \frac{\partial\omega}{\partial k_x} = \omega'(k)\cos\theta = \frac{k}{\omega}\left[\frac{1}{(1+k^2)^2}+\gamma\sigma\right]\cos\theta;$$

V is, therefore, the group velocity in the modulation (x-) direction.

2.3 Derivation of the nonlinear Schrödinger equation

Proceeding to order $\sim\epsilon^3$, the equations for $l = 1$ yield an explicit compatibility condition in the form of the nonlinear Schrödinger equation (NLSE)

$$i\frac{\partial\psi}{\partial\tau}+P\frac{\partial^2\psi}{\partial\zeta^2}+Q|\psi|^2\psi = 0 \quad (4)$$

describing the evolution of the potential perturbation $\psi \equiv \phi_1^{(1)}$. The "slow" variables $\{\zeta,\tau\}$ were defined above.

The *dispersion coefficient* P is related to the curvature of the dispersion curve as $P = (1/2)(\partial^2\omega/\partial k_x^2) = (1/2)\left[\omega''(k)\cos^2\theta+\omega'(k)(\sin^2\theta)/k\right]$; the exact form of P reads

$$P(k) = \frac{1}{\beta}\frac{1}{2\omega}\left(\frac{\omega}{k}\right)^4\left[\nu_1-\left(\nu_1+3\frac{\nu_2}{\beta}\omega^2\right)\cos^2\theta\right], \quad (5)$$

where we have defined:

$$\nu_1 = \beta\frac{\beta+\sigma\gamma(1+k^2)^2}{[\beta+\sigma\gamma(1+k^2)]^2}$$

and

$$\nu_2 = \beta^3\frac{3\beta+\gamma\sigma(3-k^2)(1+k^2)}{3[\beta+\gamma\sigma(1+k^2)]^4}$$

(see that $\nu_{1,2}\to 1$ when $\sigma\to 0$, recovering the previous cold ion-model result [30]).

The *nonlinearity coefficient* Q is due to carrier wave self-interaction. Distinguishing different contributions, Q can be split into five distinct parts, viz. $Q = \sum_{j=0}^{4} Q_j$, where $Q_{0/2}$ is due to the zeroth/second order harmonics and Q_1 is related to the cubic term in (1):

see equations (6–8) above

while $Q_{3,4}$ (too lengthy to report here) are related to ion pressure via σ (and cancel in the "cold-ion" model, i.e. for $\sigma = 0$). In fact, $Q_{1,2}$ are isotropic, while $Q_{0,3,4}$ depend on the angle θ.

Notice the influence on the form (and the sign) of coefficients P and Q of: (a) the modulation angle θ (as compared to the parallel DIAW modulation case; cf. Ref. [24]), (b) the dust component density n_d via β defined above (as compared to the IAW case in an electron-ion plasma; cf. Refs. [16,19]) and (c) the ion temperature via σ (as compared to the cold-ion DP models; cf. Refs. [26,30]). The role of the former two was exhaustively investigated in reference [30] — so those results will not be reproduced here — while the latter effect (temperature) will be studied in the remaining part of the paper. One may readily check, after a tedious yet straightforward calculation, that expressions (6) and (8) reproduce (53) and (54) in reference [26], upon setting $\sigma = 0$. However, the remaining coefficient Q_1, defined by (7), was absent therein, and yet seems to yield a rather non-negligible influence on the numerical value of Q.

An interesting result is obtained by considering the vanishing $k \ll 1$ (continuum) limit in the above formulae. As a matter of fact, both P and Q, given by

$$P\big|_{\theta=0} \approx -\frac{3}{2}\frac{\beta}{\sqrt{\beta+\gamma\sigma}}k$$

and

$$Q\big|_{\theta=0} \approx +\frac{1}{12\beta^3}\frac{1}{\sqrt{\beta+\gamma\sigma}}\left[\beta(3-2\alpha\beta)+2\gamma\sigma\right] \times \left[\beta(3-2\alpha\beta)+\gamma(\gamma+1)\sigma\right]\frac{1}{k}$$

for $\theta = 0$, change sign as oblique modulation is 'switched on':

$$P\big|_{\theta\neq 0} \approx \frac{\sqrt{\beta+\gamma\sigma}}{2k}\sin^2\theta$$

and

$$Q\big|_{\theta\neq 0} \approx -\frac{1}{12\beta^3}\frac{1}{\sqrt{\beta+\gamma\sigma}}\left[\beta(3-2\alpha\beta)+2\gamma^2\sigma\right] \times \left[\beta(3-2\alpha\beta)+\gamma(\gamma+1)\sigma\right]\frac{1}{k}.$$

One may check that the product PQ remains negative for small k, ensuring, as we shall see in the following, stability for long wavelengths $\lambda \gg \lambda_D$.

In conclusion, both coefficients P and Q may change sign when "switching on" *theta*. Obliqueness in modulation therefore affects the stability profile of the system, which confirms the general qualitative arguments put forward in reference [19] for the ion-acoustic wave in an electron-ion plasma (without dust). Nevertheless, at all cases, the product of P and Q is negative for small k, ensuring, as we shall see, stability for long carrier wavelengths. This qualitative remark, which remains true for any value of β (i.e. regardless of the concentration of the dust component) is in complete agreement with previous results in the ion-acoustic wave case (with no dust): see e.g. equation (41) in reference [16]; notice, in passing, that the result $Q \sim 1/k$ obtained therein is also recovered here [34].

3 Stability analysis

The NLSE (4) is known to possess the monochromatic (Stokes wave) solution: $\psi = \hat{\psi}e^{iQ|\hat{\psi}|^2\tau} + c.c.$ We may consider small perturbations by setting $\hat{\psi} = \hat{\psi}_0 + \epsilon\hat{\psi}_1$, and then take $\hat{\psi}_1$ to be of the form: $\hat{\psi}_1 = \hat{\psi}_{1,0}e^{i(\hat{k}\zeta-\hat{\omega}\tau)} + c.c.$ (the perturbation wavenumber $\hat{k}$ and frequency $\hat{\omega}$ should be distinguished from the carrier wave quantities, k and ω). Now, substituting into (4), we readily obtain the perturbation dispersion relation

$$\hat{\omega}^2 = P^2\hat{k}^2\left(\hat{k}^2 - 2\frac{Q}{P}|\hat{\psi}_0|^2\right).$$

The wave will be *stable* for all values of $\hat{k}$ if the product PQ is negative. However, for positive $PQ > 0$, instability sets in for wavenumbers below a critical value $\hat{k}_{cr} = \sqrt{2Q/P}|\hat{\psi}_0|$, i.e. for wavelengths above a threshold: $\lambda_{cr} = 2\pi/\hat{k}_{cr}$; defining the instability growth rate $\sigma = |\mathrm{Im}\hat{\omega}(\hat{k})|$, we see that it reaches its maximum value for $\hat{k} = \hat{k}_{cr}/\sqrt{2}$, viz. $\sigma_{\max} = |\mathrm{Im}\hat{\omega}|_{\hat{k}=\hat{k}_{cr}/\sqrt{2}} = |Q||\hat{\psi}_0|^2$. In brief, we see that the instability condition depends only on the sign of the product PQ, which can now be studied numerically, relying on the exact expressions derived in the preceding section.

In the contour plots presented below (see Figs. 1 to 3), we have depicted the $PQ = 0$ boundary curve against

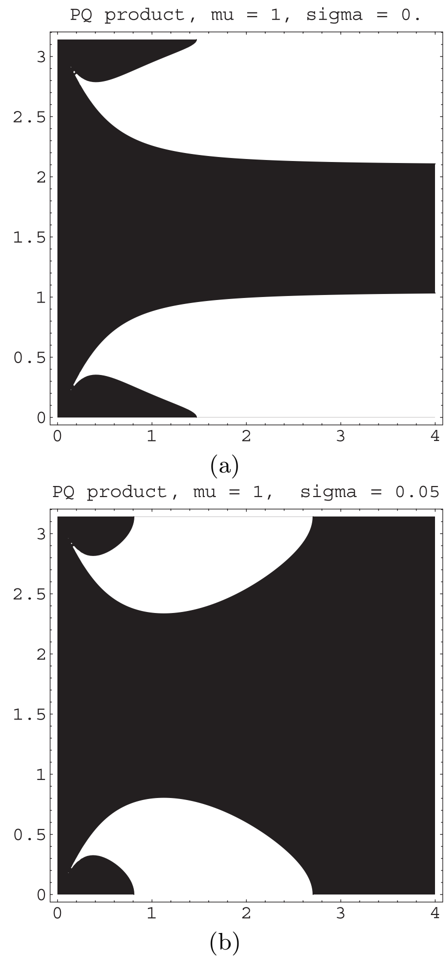

Fig. 1. The $PQ = 0$ curve is represented against normalized wavenumber k/k_D (horizontal axis) and angle θ (vertical axis); the area in black (white) represents the region in the $(k-\theta)$ plane where the product is negative (positive), i.e. where the wave is stable (unstable). This plot refers to the *dust-free* case ($\delta = 0$ i.e. $\mu = 1$). (a) $\sigma = 0$ (cold ions); (b) $\sigma = 0.05$ (warm ions).

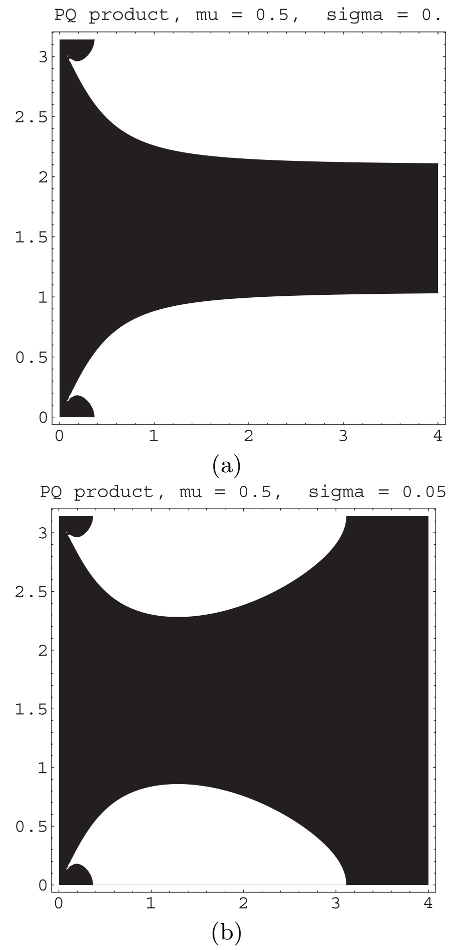

Fig. 2. Similar to the previous figure, for negative dust charge; we have taken a *negative* dust density: $\delta = q_{d,0}/q_{i,0} = 0.5$ i.e. $\mu = 0.5$ here.

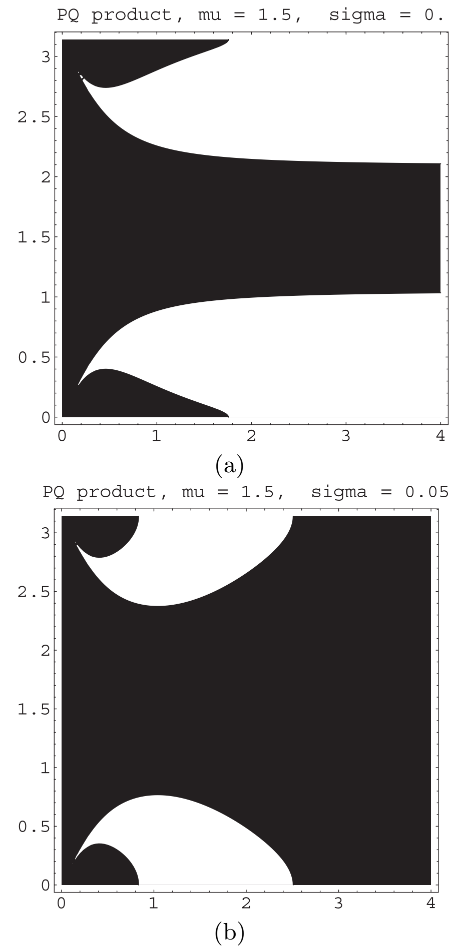

Fig. 3. Similar to the previous figures, for positive dust charge; we have taken a *positive* dust density: $\delta = q_{d,0}/q_{i,0} = 0.5$ i.e. $\mu = 1.5$ here.

the normalized wavenumber k/k_D and angle θ (between 0 and π; notice the $\pi/2$-periodicity); the area in black (white) represents the region in the $(k-\theta)$ plane where the product is negative (positive) i.e. where the carrier wave will be stable (unstable) to external perturbations. We have taken $Z_i = 1$ (implying: $\alpha = 1/2$, $\alpha' = 1/6$ and $\beta = 1/\mu$), and then considered different values of μ and σ in the plots.

For a given (fixed) value of angle θ, instability sets in at some critical value of the wavenumber $k_{cr,1}$; therefore, waves characterized by a wavenumber lower than $k_{cr,1}$ (or wavelength higher than $\lambda_{cr,1} = 2\pi/k_{cr,1}$) will be *stable*. For the sake of reference, note that the known values of $k_{cr,1} = 1.47k_D$ *in the absence of dust*, and for parallel modulation ($\theta = 0$) [17,18], is well recovered here. As anticipated, modulation obliqueness affects the stability profile dramatically: as observed in the contour plots, and already discussed in reference [30] for the cold-ion model ($\sigma = 0$), $k_{cr,1}$ becomes lower as θ increases from zero up to a certain value (for instance, around 20 degrees in Fig. 1) and then increases to infinity for higher θ (thus prescribing stability to wide-angle modulation). On the other hand, the value of $k_{cr,1}$ is quite sensitive to changes in the dust concentration: the existence of negative (positive) dust shifts $k_{cr,1}$ towards lower (higher) values, thus enhancing (impeding) instability at long wavelengths; also see in reference [30] for details.

As far as the effect of finite ion temperature is concerned, we encounter the appearance of a second finite threshold $k_{cr,2}$ as σ is switched on (i.e. $k_{cr,2} \to \infty$ for $\sigma \to 0$); very short wavelengths are therefore stable as well. It should be pointed out that this result may be of importance in dusty plasmas, as DP is known *not* to be subject to Landau damping [1] (which restricts the validity of results for short wavelengths), contrary to "ordinary" (dust-free) an electron-ion plasma. As the ion temperature gradually increases, both thresholds $k_{cr,1}$ and $k_{cr,2}$ are shifted and the instability region gradually shrinks; cf. Figures 1, 2, 3 (a to b); also see Figure 4. Notice, however, the qualitatively different behaviour between negative dust and positive dust: the existence of the former ($q_d < 0$) results in a slightly wider (as compared to the dust-free case) region of instability, while the latter ($q_d > 0$) does exactly the opposite (yet the effect is less intense). Positive dust, therefore, seems to slightly enhance stability, in agreement to results in reference [30]. In any case, the ion

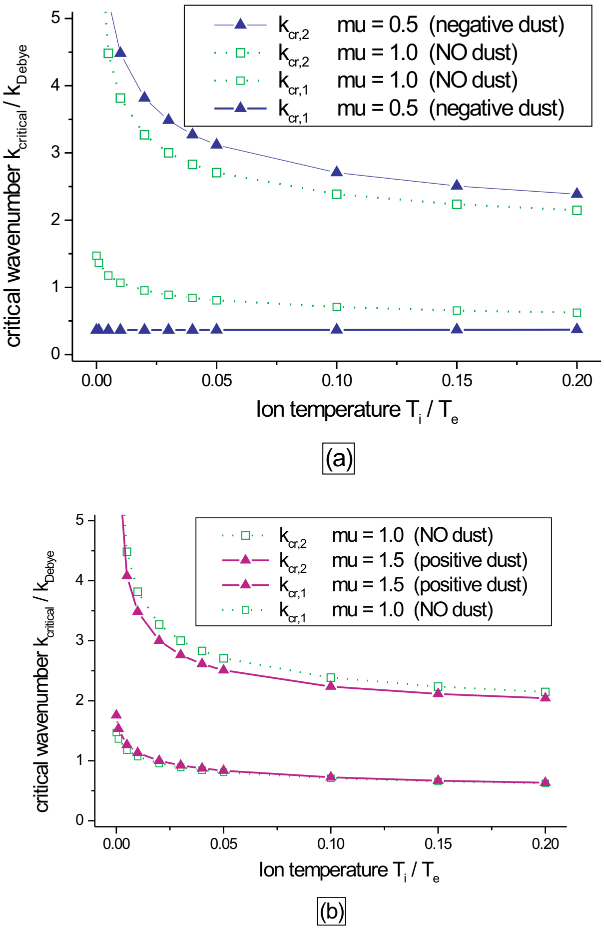

Fig. 4. The two critical wavenumber values $k_{cr,1}/k_{cr,2}$ are depicted (lower/upper solid line, triangles) against normalized ion temperature $\sigma = T_i/T_e$, for a dusty plasma with negative dust charge. The unstable region lies between the two curves. The dust-free case $k_{cr,1}/k_{cr,2}$ thresholds (lower/upper dot line, squares) is also depicted, for reference. (a) *Negative* dust: $\delta = q_{d,0}/q_{i,0} = 0.5$ i.e. $\mu = 0.5$; (b) *positive* dust: $\delta = 0.5$ i.e. $\mu = 1.5$. See that the addition of negative (positive) dust results in a wider (narrower) instability region, which gets even narrower with temperature, in both case.

temperature influences the stability profile quite strongly, in qualitative agreement with experiments [35].

4 Envelope excitations

The NLS equation (4) is known to possess several types of localized solutions (envelope solitons), representing propagating constant profile perturbations of the carrier envelope; these should be distinguished from non-topological soliton solutions — pulses — or shocks, obtained for the same physical system via a different formalism [39]). Following closely references [30,40,41], we shall only briefly review the analytical form of solutions which are of relevance to our problem (the different types of NLS solutions are exhaustively reviewed, e.g. in Ref. [41]).

Seeking a solution to equation (4) in the form $\psi(\zeta,\tau) = \sqrt{\rho(\zeta,\tau)}e^{i\Theta(\zeta,\tau)}$, where ρ, σ are real variables to be determined, one obtains the following (envelope) solutions:

(a) the (bright) envelope soliton [42], for $PQ > 0$:

$$\rho = \rho_0 \mathrm{sech}^2\left(\frac{\zeta - u\tau}{L}\right),$$
$$\Theta = \frac{1}{2P}\left[u\zeta - \left(\Omega + \frac{1}{2}u^2\right)\tau\right], \qquad (9)$$

representing a localized pulse travelling at a speed u and oscillating at a frequency Ω (at rest). The pulse width L depends on the (constant) maximum amplitude square ρ_0 as $L = \sqrt{2P/Q\rho_0}$. Note that this solution, when the envelope width is close to the carrier wavelength, is the continuum analogue of the (discrete) "breather" modes studied in molecular chains [43];

(b) the *dark* envelope soliton (*hole*) [42], for $PQ < 0$:

$$\rho = \rho_1\left[1 - \mathrm{sech}^2\left(\frac{\zeta - u\tau}{L'}\right)\right] = \rho_1 \tanh^2\left(\frac{\zeta - u\tau}{L'}\right),$$
$$\Theta = \frac{1}{2P}\left[u\zeta - \left(\frac{1}{2}u^2 - 2PQ\rho_1\right)\tau\right], \qquad (10)$$

representing a localized region of negative wave density (hole) travelling at a speed u; again, the pulse width depends on the maximum amplitude square ρ_1 via $L' = \sqrt{2\left|P/Q\rho_1\right|}$;

(c) the *grey* envelope solitary wave, also obtained for $PQ < 0$ [41]:

$$\rho = \rho_2\left[1 - a^2\mathrm{sech}^2\left(\frac{\zeta - u\tau}{L''}\right)\right],$$
$$\Theta = \frac{1}{2P}\left[V_0\zeta - \left(\frac{1}{2}V_0^2 - 2PQ\rho_2\right)\tau + \Theta_{10}\right] - S\sin^{-1}\frac{a\tanh\left(\frac{\zeta-u\tau}{L''}\right)}{\left[1 - a^2\mathrm{sech}^2\left(\frac{\zeta-u\tau}{L''}\right)\right]^{1/2}}, \qquad (11)$$

which also represents a localized region of negative wave density; Θ_{10} is a constant phase; S denotes the product $S = \mathrm{sign}\,P \times \mathrm{sign}\,(u - V_0)$. In comparison to the dark soliton (10), note that apart from the maximum amplitude ρ_2, which is now finite (i.e. non-zero) everywhere, the pulse width of this grey-type excitation: $L'' = \sqrt{2\left|P/Q\rho_2\right|}/a$, now also depends on a, given by:

$$a^2 = 1 + \frac{1}{2PQ}\frac{1}{\rho_2}\left(u^2 - V_0^2\right) \le 1,$$

an independent parameter representing the modulation depth ($0 < a \le 1$). V_0 is an independent real

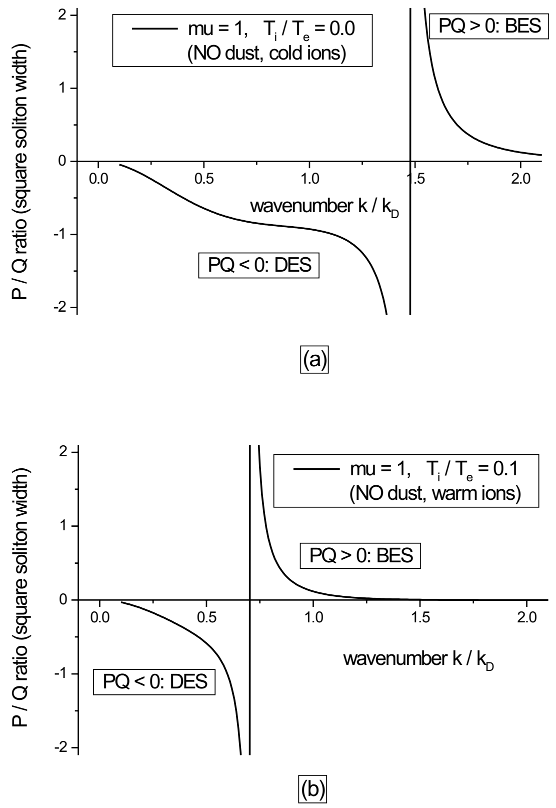

Fig. 5. The P/Q ratio, whose absolute value/sign is related to the (square) soliton width/type (see in the text), is plotted against the wavenumber k, for parallel modulation ($\theta = 0$). This plot refers to the *dust-free* case ($\delta = 0$ i.e. $\mu = 1$). (a) $\sigma = 0$ (cold ions); (b) $\sigma = 0.05$ (warm ions). Positive/negative values correspond to BES/DES: bright/dark envelope solitary waves.

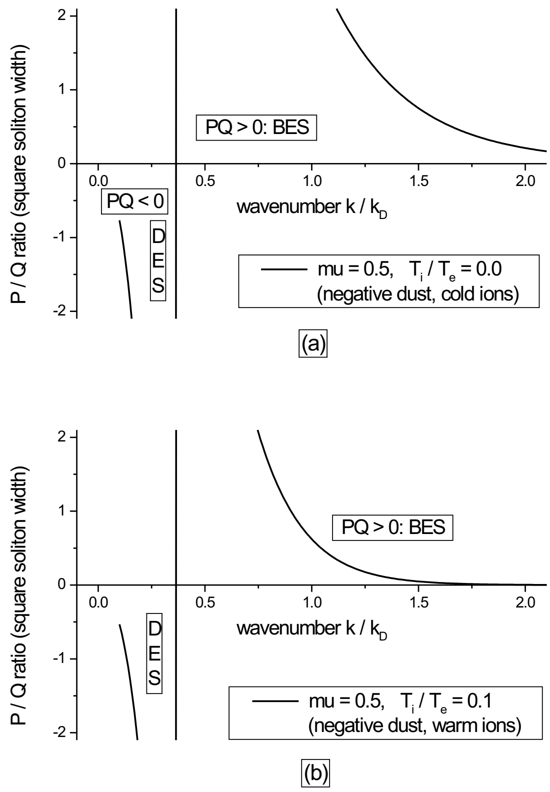

Fig. 6. Similar to the previous figure, for negative dust charge; we have taken a *negative* dust density: $\delta = q_{d,0}/q_{i,0} = 0.5$ i.e. $\mu = 0.5$ here.

constant which satisfies [41]: $V_0 - \sqrt{2|PQ|\rho_2} \leq u \leq V_0 + \sqrt{2|PQ|\rho_2}$ (for $V_0 = u$, we have $a = 1$, and thus recover the *dark* soliton above).

We see that the regions depicted in Figures 1–3 in fact also distinguish the regions where different types of localized solutions may exist: bright (dark or grey) solitons will occur in white (black) regions. One immediately draws the conclusion that the ion temperature may strongly affect the type of solitary waves sustained in the system, e.g. destabilizing bright-type modes (pulses) and favoring dark ones (holes), or vice versa. Furthermore, the soliton characteristics (width, amplitude) will depend on temperature via (the appearance of σ in) the P and Q coefficients; for instance, regions with higher values of P (or lower values of Q) — see Figures 5 to 7 — will support wider (i.e. spatially more extended) localized excitations. As a matter of fact, increasing the ion temperature has a multifold effect: it destabilizes dark excitations at long wavelengths, in favor of bright ones, and it supports slightly narrower excitations of either kind, while is also allows for dark envelopes at high k (short wavelength); cf. e.g. Figures 1a to 1b, and so forth. Also notice that, at a given temperature, the addition of negative (positive) dust to the plasma apparently results in higher (lower) values of P/Q i.e. wider (narrower) envelope excitations, for a given amplitude (see e.g. Figs. 5 to 7).

5 Conclusions

This work was dedicated to the study of the modulational instability of dust-ion acoustic waves propagating in an unmagnetized dusty plasma. Summarizing our results, we have seen that

(i) obliqueness in modulation may strongly affect the conditions for modulational instability to occur: regions which are stable to parallel modulation may become unstable when subject to oblique modulation, and vice versa;
(ii) large-angle modulation seems to have a stabilizing effect; on the contrary, small-to-medium angle modulation (for k between two critical values, depending on the dust concentration) enhances instability; however, this is suppressed by taking into account ion temperature, which appears to have a stabilizing effect on short-wavelength DIA waves;

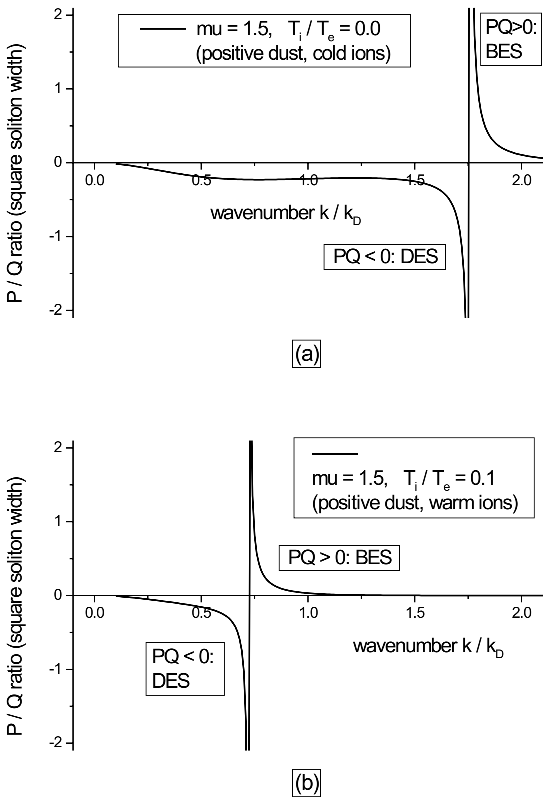

Fig. 7. Similar to the previous two figures, for positive dust charge; we have taken a *positive* dust density: $\delta = q_{d,0}/q_{i,0} = 0.5$ i.e. $\mu = 1.5$ here.

(iii) DIAW-related localized envelope excitations (solitary waves) may be formed and sustained in a dusty plasma; stable (unstable) DIA wave regions in the (k, θ) plane support envelope solitary waves of the bright (dark or grey) type, for which explicit expressions are known. Furthermore, the type and characteristics of these localized modes depend strongly on the ion temperature — which seems to favour dark-type excitations (holes) — as well as the modulation angle θ, dust concentration n_d and dust sign s.

These results confirm and complete the qualitative conclusions drawn from the cold-ion model [30]. Our aim here was to suggest a fully three-dimensional model for DIAW modulation in unmagnetized collisionless dusty plasma which is generic, by incorporating features like obliqueness in modulation, negative or positive dust charge, arbitrary carrier wavelength λ (remember that short λ electrostatic plasma waves are subject to Landau damping and thus often excluded from the analysis; this is not the case in DP) and finite ion temperature. However, we have assumed dust charge to be constant and the plasma geometry was taken to be Cartesian and infinite, for simplicity. These effects may be included in a forthcoming work.

This work was supported by the European Commission (Brussels) through the Human Potential Research and Training Network via the project entitled: "Complex Plasmas: The Science of Laboratory Colloidal Plasmas and Mesospheric Charged Aerosols" (Contract No. HPRN-CT-2000-00140).

References

1. P.K. Shukla, A.A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics Publishing Ltd., Bristol, 2002)
2. N.N. Rao, P.K. Shukla, M.Y. Yu, Planet. Space Sci. **38**, 543 (1990)
3. P.K. Shukla, V.P. Silin, Phys. Scripta **45**, 508 (1992)
4. A. Barkan, R. Merlino, N. D'Angelo, Phys. Plasmas **2**, 3563 (1995)
5. J. Pieper, J. Goree, Phys. Rev. Lett. **77**, 3137 (1996)
6. A. Barkan, N. D'Angelo, R. Merlino, Planet. Space Sci. **44**, 239 (1996)
7. F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer Academic Publishers, Dordrecht, 2001)
8. N.A. Krall, A.W. Trivelpiece, *Principles of plasma physics* (McGraw-Hill, New York, 1973)
9. Th. Stix, *Waves in Plasmas* (American Institute of Physics, New York, 1992)
10. T. Taniuti, N. Yajima, J. Math. Phys. **10**, 1369 (1969)
11. N. Asano, T. Taniuti, N. Yajima, J. Math. Phys. **10**, 2020 (1969)
12. A.S. Davydov, *Solitons in Molecular Systems* (Kluwer Academic Publishers, Dordrecht, 1985)
13. A. Hasegawa, *Optical Solitons in Fibers* (Springer-Verlag, 1989)
14. E. Infeld, G. Rowlands, *Nonlinear Waves, Solitons and Chaos* (Cambridge University Press, Cambridge, England, 1990)
15. M. Remoissenet, *Waves Called Solitons* (Springer-Verlag, Berlin, 1994)
16. K. Shimizu, H. Ichikawa, J. Phys. Soc. Jpn **33**, 789 (1972)
17. M. Kako, Prog. Theor. Phys. Suppl. **55**, 1974 (1974)
18. T. Kakutani, N. Sugimoto, Phys. Fluids **17**, 1617 (1974)
19. M. Kako, A. Hasegawa, Phys. Fluids **19**, 1967 (1976)
20. R. Chhabra, S. Sharma, Phys. Fluids **29**, 128 (1986)
21. M. Mishra, R. Chhabra, S. Sharma, Phys. Plasmas **1**, 70 (1994)
22. V. Chan, S. Seshadri, Phys. Fluids **18**, 1294 (1975)
23. I. Durrani et al., Phys. Fluids **22**, 791 (1979)
24. J.-K. Xue, W.-S. Duan, L. He, Chin. Phys. **11**, 1184 (2002)
25. This remark excludes e.g. the electron plasma mode, which was found to be stable to parallel perturbations for all carrier wavelengths; see in references [11,18]
26. M.R. Amin, G.E. Morfill, P.K. Shukla, Phys. Rev. E **58**, 6517 (1998)
27. R.-A. Tang, J.-K. Xue, Phys. Plasmas **10**, 3800 (2003)
28. I. Kourakis, P.K. Shukla, J. Math. Phys. (2003, submitted)
29. Xue Jukui, Lang He, Phys. Plasmas **10**, 339 (2003)
30. I.Kourakis, P.K. Shukla, Phys. Plasmas **10**, 3459 (2003)
31. M.R. Amin, G.E. Morfill, P.K. Shukla, Phys. Plasmas **5**, 2578 (1998); Phys. Scripta **58**, 628 (1998)
32. I. Kourakis, *Proceedings of the 29th EPS meeting on Controlled Fusion and Plasma Physics, European Conference Abstracts (ECA)*, Vol. 26B P-4.221 (European Physical Society, Petit-Lancy, Switzerland, 2002)

33. Notice that the parameters α, α', β all take positive values of similar order of magnitude (for $Z_i = 1$) and may not be neglected; see, for instance, in reference [26] where α' is omitted
34. As a matter of fact, the numerical factor 1/3 in reference [16] is exactly obtained here upon setting $\alpha = 1/2$, $\alpha' = 1/6$, $\beta = 1$ and $\sigma = 0$, into our formulae
35. W. Watanabe, J. Plasma Phys. **17**, 487 (1977)
36. W. Watanabe, J. Plasma Phys. **14**, 353 (1975)
37. Q.-Z. Luo, N. D'Angelo, R. Merlino, Phys. Plasmas **5**, 2868 (1998)
38. Y. Nakamura, H. Bailung, P.K. Shukla, Phys. Rev. Lett. **83**, 1602 (1999); Y. Nakamura, A. Sarma, Phys. Plasmas **8**, 3921 (2001)
39. P.K. Shukla, Phys. Plasmas **10**, 1619 (2003)
40. A. Hasegawa, *Plasma Instabilities and Nonlinear Effects* (Springer-Verlag, Berlin, 1975)
41. R. Fedele, H. Schamel, Eur. Phys. J. B **27**, 313 (2002); R. Fedele, H. Schamel, P.K. Shukla, Phys. Scripta T **98**, 18 (2002)
42. This expression is readily obtained from reference [41], by shifting the variables therein to our notation as: $x \to \zeta$, $s \to \tau$, $\rho_m \to \rho_0$, $\alpha \to 2P$, $q_0 \to -2PQ$, $\Delta \to L$, $E \to \Omega$, $V_0 \to u$.
43. S. Flach, C. Willis, Phys. Rep. **295**, 181 (1998); also see several papers in the volume: *Fluctuations, Disorder and Nonlinearity*, edited by G. Tsironis, E.N. Economou, Physica D **113** (North-Holland, Amsterdam, 1998)