Modulated Wave Packets and Envelope Solitary Structures in Complex Plasmas

Ioannis Kourakis and Padma K. Shukla

***Abstract*—A theoretical study is presented of the nonlinear amplitude modulation of waves propagating in unmagnetized plasmas contaminated by charged dust particles. Distinct well-known dusty plasma modes are explicitly considered, namely, the dust-acoustic wave, the dust-ion acoustic wave, and transverse dust-lattice waves. Using a multiple-scale technique, a nonlinear Schrödinger-type equation is derived, describing the evolution of the wave amplitude. A stability analysis reveals the possibility for modulational instability to occur, possibly leading to the formation of different types of envelope-localized excitations (solitary waves), under conditions which depend on the wave dispersion laws and intrinsic dusty plasma parameters.**

***Index Terms*—Complex plasma, electrostatic plasma waves, envelope solitons, modulational instability, nonlinear Schrödinger (NLS) equation.**

I. Introduction

LARGE ensembles of charged particles (plasmas) contaminated with massive heavily charged dust particles, known as complex or dusty plasmas (DP), have recently attracted a constantly increasing interest among researchers, due to their wide occurrence in space and laboratory plasmas and the wealth of novel physics involved in their description [1], [2]. The characteristics of known harmonic modes [3], [4] propagating in a plasma are strongly modified by the presence of dust [1], [2]; for instance, the frequency range of ion-acoustic waves (IAW) [3], [4] is strongly modified in the presence of dust, giving rise to dust-ion acoustic waves (DIAW) [5]. Furthermore, brand new low-frequency modes have been shown to exist in dusty plasma, either as electrostatic oscillations related to dust grain inertia, in weakly coupled ("gaseous") complex plasmas, e.g., the dust-acoustic wave (DAW) [1], [6]–[8], or as periodic excitations propagating in strongly coupled ("crystalline phase") DP arrangements ("quasi-lattices"), e.g., dust-lattice waves (DLW) [1], [9]–[14]. Interestingly, the phase velocity of these dusty plasma modes lies far from both electron and ion thermal velocities $v_{\mathrm{th},e/i} = (T_{e/i}/m_{e/i})^{1/2}$, thus ruling out the occurrence of Landau damping, known to prevail over electrostatic wave propagation in electron–ion (e–i) plasma [1]–[4].

It is now commonly known that linear oscillations, yet indispensable as a first approach to most dynamical problems, are rarely left to lead a quiet undisturbed life. Once the slightest important departure from equilibrium is considered, omnipresent nonlinearity mechanisms come into play, initially manifesting themselves via harmonic generation due to carrier wave self-modulation, at a first step, then possibly inducing instability to external perturbations, and eventually leading to wave mode breakup (collapse); in many situations, the interplay between nonlinearity and wave dispersion may lead to the formation of nonequilibrium stationary localized structures. This generic picture is known to be valid in physical contexts as diverse as hydrodynamics, condensed matter, nonlinear optics, signal transmission lines, etc. [16], [17].

Plasma physics has, rather not surprisingly, always provided an excellent test-bed for nonlinear modulation theories [18]–[31] and dusty plasma has been no exception [32]–[39], [56]. Reviewing a few noteworthy results, electron plasma modes have been shown to be stable against parallel modulation [20]; the ion plasma modes do as well, yet only for perturbations below a specific wavenumber threshold [24]. Ion acoustic modes are stable to parallel modulation [18]–[22] for wavelengths above a specific threshold, which is seen to increase if one takes into account finite-temperature effects [23]–[25] or oblique amplitude modulation [26]–[28]. These results have been confirmed by kinetic-theoretical studies [29]–[31], for ion–acoustic waves in e–i plasmas. In dusty plasma, the amplitude modulation of the DP acoustic modes has been investigated in [32]–[36]; similar studies have been carried out for oscillations in DP crystals [37]–[39], [56].

This paper aims at investigating the role of nonlinear amplitude modulation mechanism in dusty plasma modes, first, and also questioning, in a generic manner, the existence of envelope excitations (solitons) related to these modes. By making use of a multiple space and time scale technique [18], [19], a nonlinear Schrödinger-type equation (NLSE) is derived, describing the evolution of the wave envelope. The conditions for modulational instability are discussed, pointing out their dependence on the carrier wavenumber k, the modulation angle θ, and dust concentration n_d. Finally, the existence of different types of envelope dusty plasma solitons is anticipated. In order to keep a low level of complexity in this qualitative description, known DP dissipative mechanisms like collisions with neutrals, ion drag, and dust charge variations are omitted; these effects are left for consideration in forthcoming work.

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I. Kourakis is with the Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany, and also with Faculté des Sciences Apliquées, Université Libre de Bruxelles (U.L.B.), B-1050 Brussels, Belgium (e-mail: ioannis@tp4.rub.de).

P. K. Shukla is with the Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany.

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II. Dust–Acoustic Waves

The DAW, theoretically predicted in [6] and later experimentally observed [7], [8], is a very low-frequency purely DP mode (e.g., absent without dust) representing inertial dust grain oscillations against a thermalized background of electrons and ions, which provide the necessary restoring force. The DAW phase velocity is much smaller than both electron and ion thermal speeds and its frequency is below the dust plasma frequency $\omega_{p,D}$.

A. Model Equations

In order to formulate a simple model for DAW propagation, we consider a three component collisionless unmagnetized dusty plasma consisting of electrons (mass m, charge e), ions (mass m_i, charge $q_i = +Z_i e$), and heavy dust particulates (mass m_d, charge $q_d = sZ_d e; s = \mathrm{sgn} q_d = \pm 1$), henceforth denoted by e, i, d, respectively. Dust mass and charge will be taken to be constant, for simplicity.

The dust component moment-Poisson system of (reduced) evolution equations is

$$\begin{aligned} &\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0, \\ &\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -s\nabla\phi - \frac{\sigma}{n}\nabla p, \\ &\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p = -\gamma p \nabla \cdot \mathbf{u} \end{aligned} \tag{1}$$

where all quantities are dimensionless; $n, \mathbf{u}$ and p, respectively, denote the (normalized) dust density $n_d/n_{d,0}$ and mean velocity $\mathbf{u}_d/v_d = [m_d/(k_B T_e)]^{1/2}\mathbf{u}_d$, and pressure $p_d/p_0 = p_d/(n_{d,0}k_B T_d); \gamma = (f+2)/f$ is the ratio of specific heats (f is the number of degrees of freedom). For example, $\gamma = 3$ in the adiabatic one-dimensional (1-D) case; space and time are scaled over the DP effective Debye length $\lambda_{\mathrm{D,eff}} = (\lambda_{D,e}^{-2} + \lambda_{D,i}^{-2})^{-1/2}$ (where $\lambda_{D,\alpha} = (k_B T_\alpha/4\pi n_{\alpha,0} q_\alpha^2)^{1/2}, \alpha = e, i$) and the inverse DP plasma frequency $\omega_{p,d}^{-1} = (4\pi n_{d,0} q_d^2/m_d)^{-1/2}$, respectively. The (reduced) electric potential $\phi = Z_d e\Phi/(k_B T_e)$ obeys Poisson's equation: $\nabla^2\Phi = -4\pi \sum q_\alpha n_\alpha$, which here it takes the form

$$\nabla^2\phi = \phi - \alpha\phi^2 + \alpha'\phi^3 - s\beta(n-1) \tag{2}$$

by linearizing around the Maxwellian state assumed for both electrons and ions, e.g., $n_e \approx n_{e,0}e^{e\Phi/k_B T_e}, n_i \approx n_{i,0}e^{-Z_i e\Phi/k_B T_i}; T_\alpha$ is the temperature of species $\alpha = e, i$ (k_B is the Boltzmann constant). The dimensionless parameters are: $\alpha = (1/2Z_d)[Z_i^3(T_e/T_i)^2 n_{i,0}/n_{e,0} - 1]/(Z_i^2(T_e/T_i)n_{i,0}/n_{e,0}) + 1)$, $\alpha' = (1/6Z_d^2)[Z_i^4(T_e/T_i)^3(n_{i,0}/n_{e,0}) + 1]/(Z_i^2)(T_e/T_i)n_{i,0}/n_{e,0}) + 1)$, and $\beta = (\lambda_{D,\mathrm{eff}}\omega_{p,d}/v_d)^2 \equiv (c_D/v_d)^2$, where $c_D = \lambda_{D\mathrm{eff}}\omega_{p,d}$ is the DA speed [1]. Alternatively, one may define these parameters in terms of typical dust parameters, e.g., either the ratio $\mu = n_{e,0}/(Z_i n_{i,0})$ or $\delta = (Z_d n_{d,0})/(Z_i n_{i,0})$. For $\mu \ll Z_i T_e/T_i$, one has $\alpha \approx (Z_i/2Z_d)(T_e/T_i), \alpha' \approx (Z_i^2/6Z_d^2)(T_e^2/T_i^2) = 2/3\alpha^2$ and $\beta \approx (Z_d^2/Z_i^2)(n_{d,0}/n_{i,0})(T_i/T_e)$. Since overall neutrality is assumed at equilibrium $n_{e,0} - Z_i n_{i,0} - sZ_d n_{d,0} = 0$, one has $\mu = 1 + s\delta$ so that $0 \le \mu < 1$ ($\mu > 1$) corresponds to negative (positive) dust charge; obviously, $\mu = 0$ in the absence of dust.

Notice that the temperature ratio $\sigma = p_0/(n_{d,0}k_B T_e) = T_d/T_e$ will "tag" the effect of the pressure evolution being taken into account in this "warm" dust model [33]. In contrast, e.g., to [36], we see that the "cold" model) expressions (9)–(11) therein are readily recovered upon setting $\sigma = 0$ (and $s = -1, \alpha' = 0$).

B. Multiple Scales Perturbation Method

Let $\mathbf{S}$ be the state (column) vector $(n, \mathbf{u}, p, \phi)^T$, describing the system's state at a given position $\mathbf{r}$ and instant t. We shall consider small deviations from the equilibrium state $\mathbf{S}^{(0)} = (1, \mathbf{0}, 1, 0)^T$ by taking $\mathbf{S} = \mathbf{S}^{(0)} + \epsilon\mathbf{S}^{(1)} + \epsilon^2\mathbf{S}^{(2)} + \cdots = \mathbf{S}^{(0)} + \sum_{n=1}^{\infty}\epsilon^n\mathbf{S}^{(n)}$, where $\epsilon \ll 1$ is a smallness parameter. Following the standard multiple scale (reductive perturbation) technique [18], [19], we shall consider the stretched (slow) space and time variables $\zeta = \epsilon(x - \lambda t), \tau = \epsilon^2 t$ ($\lambda \in \Re$). The perturbed states are assumed to depend on the fast scales via the carrier phase $\theta_1 = \mathbf{k}\cdot\mathbf{r} - \omega t$, while the slow scales enter the argument of the jth element's lth harmonic amplitude $S_{j,l}^{(n)}$, allowed to vary along x, viz., $Sj^{(n)} = \sum_{l=-\infty}^{\infty} S_{j,l}^{(n)}(\zeta,\tau)e^{il(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ (where $S_{j,-l}^{(n)} = S_{j,l}^{(n)*}$). The amplitude modulation (along the x axis) is thus allowed to take place in an oblique direction, with respect to the (arbitrary) propagation direction; accordingly, the wavenumber $\mathbf{k}$ is $(k_x, k_y) = (k\cos\theta, k\sin\theta)$. Treating the derivative operators as

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \epsilon\lambda\frac{\partial}{\partial\zeta} + \epsilon^2\frac{\partial}{\partial\tau}, \quad \nabla \to \nabla + \epsilon\hat{x}\frac{\partial}{\partial\zeta}, \quad \nabla^2 \to \nabla^2 + 2\epsilon\frac{\partial^2}{\partial x\partial\zeta} + \epsilon^2\frac{\partial^2}{\partial\zeta^2}$$

and substituting into the system of evolution equations, one obtains an infinite series in both (perturbation order) ϵ^n and (phase harmonic) l. The standard perturbation procedure now consists of solving in successive orders $\sim\epsilon^n$ and substituting in subsequent orders. The calculation, particularly lengthy yet perfectly straightforward, is reported elsewhere [33], so only the essential stepstones are provided here. The $(n = 2, l = 1)$ equations determine the first harmonics of the perturbation

$$\begin{aligned} n_1^{(1)} &= s\frac{1+k^2}{\beta}\phi_1^{(1)} = \frac{1}{\gamma}p_1^{(1)} = \frac{1}{\omega}\mathbf{k}\cdot\mathbf{u}_1^{(1)} \\ &= \frac{k}{\omega\cos\theta}u_{1,x}^{(1)} = \frac{k}{\omega\sin\theta}u_{1,y}^{(1)} \end{aligned} \tag{3}$$

and provide the compatibility condition $\omega^2 = (\beta k^2/k^2 + 1) + \gamma\sigma k^2$, which exactly recovers, by restoring dimensions, the standard DAW dispersion relation [1], [6]

$$\omega^2 = \frac{c_D^2 k^2}{1 + k^2\lambda_{D_{\mathrm{eff}}}^2} + \gamma v_{\mathrm{th},d}^2 k^2. \tag{4}$$

Proceeding in the same manner, we obtain the second-order quantities, namely the amplitudes of the second harmonics $\mathbf{S}_2^{(2)}$ and constant ("direct current") terms $\mathbf{S}_0^{(2)}$, as well as a finite contribution $\mathbf{S}_1^{(2)}$ to the first harmonics; the lengthy expressions,

omitted here for brevity, can be found in [33]. The $(n = 2, l = 1)$ equations provide the compatibility condition

$$\lambda = \frac{k}{\omega}\left[\frac{1}{(1+k^2)^2} + \gamma\sigma\right]\cos\theta.$$

λ is, therefore, the group velocity $v_g(k) = \partial\omega/\partial k_x = \omega'(k)\cos\theta$ in the modulation $(x-)$ direction.

C. Derivation of the Nonlinear Schrödinger Equation

Proceeding to order $\sim\epsilon^3$, the equations for $l = 1$ yield an explicit compatibility condition in the form of the Nonlinear Schrödinger equation

$$i\frac{\partial\psi}{\partial\tau} + P\frac{\partial^2\psi}{\partial\zeta^2} + Q|\psi|^2\psi = 0 \tag{5}$$

where ψ denotes the potential correction $\phi_1^{(1)}$. The "slow" variables $\{\zeta, \tau\}$ were defined above.

The *dispersion coefficient* P is related to the curvature of the dispersion curve as $P = (1/2)(\partial^2\omega/\partial k_x^2) = (1/2)[\omega''(k)\cos^2\theta + \omega'(k)(\sin^2\theta/k)]$; the exact form of P is

$$P(k) = \frac{1}{\beta}\frac{1}{2\omega}\left(\frac{\omega}{k}\right)^4\left[\nu_1 - \left(\nu_1 + 3\frac{\nu_2}{\beta}\omega^2\right)\cos^2\theta\right] \tag{6}$$

where we have defined

$$\nu_1 = \beta\frac{\beta + \sigma\gamma(1+k^2)^2}{[\beta + \sigma\gamma(1+k^2)]^2}$$

and

$$\nu_2 = \beta^3\frac{3\beta + \gamma\sigma(3-k^2)(1+k^2)}{3[\beta + \gamma\sigma(1+k^2)]^4}$$

(see that $\nu_{1,2} \to 1$ when $\sigma \to 0$). Notice the influence of σ on (the sign of) P.

The *nonlinearity coefficient* Q is due to carrier wave self-interaction. Distinguishing different contributions, Q can be split into five distinct parts, viz., $Q = \sum_{j=0}^{4} Q_j$, where $Q_{0/2}$ is due to the zeroth/second order harmonics and Q_1 is related to the cubic term in (2)

$$\begin{aligned} Q_0 = &+\frac{1}{2\omega}\frac{1}{\beta^2}\frac{1}{(1+k^2)^2}\frac{1}{\beta+\gamma\sigma-v_g^2} \\ &\times\Big\{\beta k^2\Big[\beta\Big[3+6k^2+4k^4+k^6 \\ &+2\alpha\beta\left(s(2k^2+3)+2\alpha v_g^2\right)\Big] \\ &+\gamma\sigma[(\gamma+1)(1+k^2)^3 \\ &+2\alpha\beta(-2\alpha\beta+s\gamma(1+k^2)^2)] \\ &+[\beta(2+4k^2+3k^4+k^6+2s\alpha\beta) \\ &+2\gamma\sigma(1+k^2)^2(1+k^2+s\alpha\beta)]\cos 2\theta\Big] \\ &+2(1+k^2)^4(\beta+\gamma\sigma)\omega^2\cos^2\theta \\ &+k(1+k^2)[\beta k^2+\omega^2(1+k^2)]\frac{v_g}{\omega} \\ &\times[\beta(1+k^2+2s\alpha\beta)+\gamma(\gamma-1)\sigma(1+k^2)^2]\cos\theta\Big\} \end{aligned} \tag{7}$$

$$Q_1 = \frac{3\alpha'\beta}{2\omega}\frac{k^2}{(1+k^2)^2} \tag{8}$$

$$\begin{aligned} Q_2 = &-\frac{1}{12\beta^3}\frac{1}{\omega}\frac{1}{k^2(1+k^2)^2} \\ &\times\{2\beta k^2[5s\alpha\beta^2(1+k^2)^2+2\alpha^2\beta^3 \\ &+2\gamma^2\sigma(1+k^2)^4(1+4k^2) \\ &+\beta(1+k^2)^3(3+9k^2+2s\alpha\gamma^2\sigma)] \\ &+(1+k^2)^3\omega^2[\beta(3+9k^2+6k^4+2s\alpha\beta) \\ &+2\gamma^2\sigma(1+k^2)^2(1+4k^2)]\} \end{aligned} \tag{9}$$

while $Q_{3,4}$ (too lengthy to report here) are related to dust pressure via σ (and cancel in the "cold" model, e.g., for $\sigma = 0$). Notice that Q_1, Q_2 are *isotropic*.

An interesting result is obtained by considering the vanishing $k \ll 1$ (continuum) limit in the above formulas. As a matter of fact, both P and Q, given by $P|_{\theta=0} \approx -3/2(\beta/\sqrt{\beta+\gamma\sigma})k$ and $Q|_{\theta=0} \approx +(1/12\beta^3)1/\sqrt{\beta+\gamma\sigma})[\beta(2s\alpha\beta+3)+2\gamma\sigma][\beta(2s\alpha\beta+3)+\gamma(\gamma+1)\sigma]1/k$ for $\theta = 0$, change sign as oblique modulation is "switched on": $P|_{\theta\neq 0} \approx (\sqrt{\beta+\gamma\sigma}/2k)\sin^2\theta$ and $Q|_{\theta\neq 0} \approx -(1/12\beta^3)1/\sqrt{\beta+\gamma\sigma}[\beta(2s\alpha\beta+3)+2\gamma^2\sigma][\beta(2s\alpha\beta+3)+\gamma(\gamma+1)\sigma]1/k$. One may check that the product PQ remains negative for small k, ensuring, as we shall see in the following, stability for long wavelengths $\lambda \gg \lambda_{D,\mathrm{eff}}$.

D. Stability Analysis

The standard stability analysis [16], [40] consists of linearizing around the monochromatic (Stokes wave) solution of the NLSE (5) $\psi = \hat{\psi}e^{iQ|\hat{\psi}|^2\tau} + c.c.$ (complex conjugate) by setting $\hat{\psi} = \hat{\psi}_0 + \epsilon\hat{\psi}_1$ and taking the perturbation $\hat{\psi}_1$ to be of the form $\hat{\psi}_1 = \hat{\psi}_{1,0}e^{i(\hat{k}\zeta - \hat{\omega}\tau)} + c.c.$. Substituting into (5), one thus readily obtains $\hat{\omega}^2 = P^2\hat{k}^2(\hat{k}^2 - 2Q/P|\hat{\psi}_{1,0}|^2)$. The wave will thus be *stable* $(\forall\hat{k})$ if the product PQ is negative. However, for positive $PQ > 0$, instability sets in for wavenumbers below a critical value $\hat{k}_{cr} = \sqrt{2Q/P}|\hat{\psi}_{1,0}|$, e.g., for wavelengths above a threshold $\lambda_{cr} = 2\pi/\hat{k}_{cr}$. Defining the instability growth rate $\sigma = |\mathrm{Im}\hat{\omega}(\hat{k})|$, we see that it reaches its maximum value for $\hat{k} = \hat{k}_{cr}/\sqrt{2}$, viz.

$$\sigma_{\max} = |\mathrm{Im}\hat{\omega}|_{\hat{k}=\hat{k}_{cr}/\sqrt{2}} = |Q||\hat{\psi}_{1,0}|^2.$$

Briefly, we see that the instability condition depends only on the sign of the product PQ, which may be studied numerically, relying on the expressions derived above; see Figs. 1 and 2 where we have chosen a set of representative values $\alpha = 5\cdot 10^{-3}, \alpha' = 2\alpha^2/3 \approx 1.6\cdot 10^{-5}$ and $\beta \approx 100$, corresponding to $Z_d/Z_i = 10^3$ and $T_e/T_i = 10$ $(\gamma = 2, \sigma = 1)$.

As predicted above, the wave is always stable for very long wavelengths $(k \ll 1)$. For parallel modulation $(\theta = 0)$, modulational instability sets in for carrier wavenumbers k beyond a critical value, say k_{cr}. For $\sigma \neq 0$, a second threshold appears, say k'_{cr}, beyond which the wave becomes stable again. Very short wavelengths are also stable in the "hot" dust model. As anticipated, obliqueness in modulation drastically modifies the stability profile. The instability threshold k_{cr} is seen to decrease as θ varies from zero to some critical value θ_{cr}; nevertheless, beyond that value (and up to $\pi/2$) the wave remains stable. This is even true for the wavenumber regions where the wave would be *unstable* to a parallel modulation. The inverse effect is also present. Even though certain k values correspond to stability for $\theta = 0$, the same modes may become unstable when

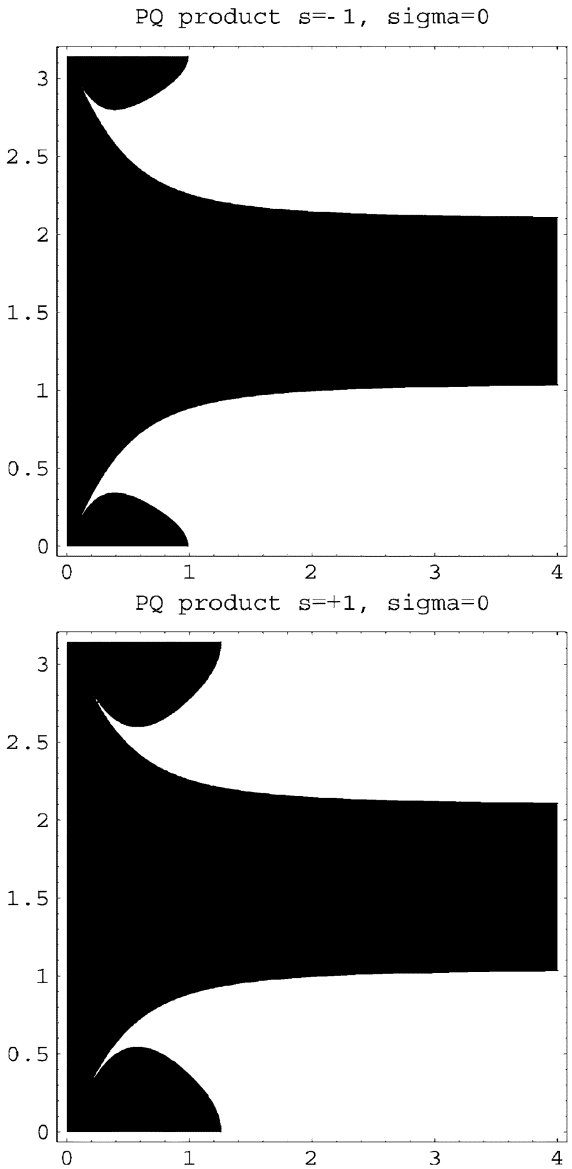

Fig. 1. DAW coefficient product $PQ = 0$ curve is represented against normalized wavenumber k/k_D (in abscissa) and angle θ (between 0 and π). Area in black (white) represents the region in the $(k - \theta)$ plane where the product is negative (positive). Instability occurs for values inside the white area. We have taken $\sigma = 0$ (cold dust): (a) negative dust charge $(s = -1)$ and (b) positive dust charge $(s = +1)$.

subject to an oblique modulation $(\theta \neq 0)$; this is mostly true for long wavelengths (small k). These qualitative results, obtained for negative dust, still hold in the case of positive dust [$s = +1$; see Figs. 1(b) and 2(b)], yet the instability region is somewhat narrower. Positive dust appears to slightly *favor* stability.

E. Nonlinear Excitations

The NLSE (5) is known to possess distinct types of localized constant profile (solitary wave) solutions. It should be stressed that these envelope structures should be distinguished in nature from soliton-type localized excitations, e.g., exact solutions of generic nonlinear equations like, e.g., the Korteweg–de Vries (KdV) equation; see the extensive discussion in [41]. Several types of such localized modes are reviewed in [41]–[43], so we need only briefly outline the method employed to derive the analytical form of those we are interested in and discuss their relevance to our problem.

Following [42] and [43], one may seek a solution of (5) in the form $\psi(\zeta,\tau) = \sqrt{\rho(\zeta,\tau)}e^{i\Theta(\zeta,\tau)}$. Different types of solutions are thus obtained, depending on the sign of the product PQ.

For $PQ > 0$, we find the *(bright) envelope soliton*

$$\rho = \rho_0 \mathrm{sech}^2\left(\frac{\zeta - u\tau}{L}\right), \quad \Theta = \frac{1}{2P}\left[u\zeta - \left(\Omega + \frac{1}{2}u^2\right)\tau\right] \tag{10}$$

representing a localized pulse travelling at a speed u and oscillating at a frequency Ω (for $u = 0$); see Fig. 4. The pulsewidth L depends on the (constant) maximum amplitude square ρ_0 as $L = \sqrt{2P/Q\rho_0}$.

For $PQ < 0$, we have the *dark* envelope soliton (*hole*)

$$\rho = \rho_1\left[1 - \mathrm{sech}^2\left(\frac{\zeta - u\tau}{L'}\right)\right] = \rho_1 \tanh^2\left(\frac{\zeta - u\tau}{L'}\right),$$
$$\Theta = \frac{1}{2P}\left[u\zeta - \left(\frac{1}{2}u^2 - 2PQ\rho_1\right)\tau\right] \tag{11}$$

representing a localized region (shock) of negative wave density (and zero amplitude) travelling at a speed u; see Fig. 5(a). Again, the pulsewidth depends on the maximum amplitude square ρ_1 via $L' = \sqrt{2|(P/Q\rho_1)|}$.

For $PQ < 0$, one also obtains the *gray* envelope solitary wave

$$\rho = \rho_2\left[1 - a^2 \mathrm{sech}^2\left(\frac{\zeta - u\tau}{L''}\right)\right],$$
$$\Theta = \frac{1}{2P}\left[V_0\zeta - \left(\frac{1}{2}V_0^2 - 2PQ\rho_2\right)\tau + \Theta_{10}\right] - S\sin^{-1}\frac{a\tanh\left(\frac{\zeta - u\tau}{L''}\right)}{\left[1 - a^2\mathrm{sech}^2\left(\frac{\zeta - u\tau}{L''}\right)\right]^{1/2}} \tag{12}$$

which represents a localized negative wave density region.[1]

Here, Θ_{10} is a constant phase; S denotes the product $S = \mathrm{sign}P \times \mathrm{sign}(u - V_0)$. Notice that not only the (now finite) maximum amplitude ρ but also the pulsewidth $L'' = \sqrt{2|(P/Q\rho_2)|}1/a$ now depends on a, given by $a^2 = 1 + (1/2PQ)1/\rho_2(u^2 - V_0^2) \leq 1$, an independent parameter representing the modulation depth $(0 < a \leq 1)$; see Fig. 5(b) (where $a = 0.6$). V_0 is a real constant which satisfies [42] and [43]. $V_0 - \sqrt{2|PQ|\rho_2} \leq u \leq V_0 + \sqrt{2|PQ|\rho_2}$; for $V_0 = u$, we have $a = 1$ and thus recover the *dark* soliton (11).

In conclusion, the wave instability (stability) regions depicted in white (black) in Figs. 1 and 2, in fact, also delimit the (k, θ) parameter pair values where bright (dark/gray) solutions, e.g., density pulses (holes) may exist. Furthermore, soliton characteristics will depend on the carrier wave dispersion via P and Q; for instance, regions with lower values of P (or higher values of Q) will support narrower excitations.

[1]These expressions are readily obtained from the ones in by transforming the variables therein into our notation as follows: $x \to \zeta, s \to \tau, \rho_m \to \rho_0, \alpha \to 2P, q_0 \to -2PQ, \Delta \to L, E \to \Omega, V_0 \to u$.

III. Dust–Ion Acoustic Mode

The DAW [1], [5] is the DP analogue of the well-known ion–acoustic electrostatic wave (IAW) [3], [4], where inertial ions oscillate against a background of thermal electrons and massive dust grains. The DIAW phase velocity $v_{\rm ph}$ lies between (and far from) the electron and ion thermal (sound) velocities, e.g., $v_{{\rm th},i} \ll v_{\rm ph} \ll v_{{\rm th},e}$. Finally, the DIAW frequency is much higher (lower) than the dust (ion) plasma frequency $\omega_{p,d}$ ($\omega_{i,d}$), typically tens of kilohertz in laboratory plasmas; therefore, on the time scale of interest, stationary dust does not participate in the wave dynamics and may be assumed immobile.

A. Model

The moment evolution equations for ions can be cast in a reduced (nondimensional) form identical to the system of (1) (for $s = 1$), by making use of an appropriate scaling [34], [35]; space and time are scaled over the electron Debye length $\lambda_{D,e} = (k_B T_e/4\pi n_{e,0} e^2)^{1/2}$ and the characteristic time-scale $\lambda_{D,e}/c_s \equiv \omega_{p,e}^{-1}(m_i/m_e)$, respectively, where $\omega_{p,e}$ obviously denotes the electron plasma frequency $\omega_{p,e} = (4\pi n_{e,0} e^2/m_e)^{1/2}$. The ion density, velocity, and pressure are normalized by: the ion equilibrium density $n_{i,0}$, the "sound speed" $c_s \equiv (k_B T_i/m_i)^{1/2}$, and $p_0 = n_{i,0} k_B T_e$, respectively. σ now denotes the temperature ratio T_i/T_e. The system is closed with Poisson's equation, whose right-hand side is Taylor-developed near equilibrium, e.g., $\Phi \approx 0$. Now, combining with the equilibrium charge neutrality requirement and normalizing the electric potential Φ over $k_B T_e/(Z_i e)$, one exactly recovers (2), upon setting $s = 1, \alpha \to -\tilde{\alpha}$ therein. The characteristic (dimensionless) parameters are $\tilde{\alpha} = 1/(2Z_i), \alpha' = 1/(6Z_i^2)$ and $\beta = Z_i^2 n_{i,0}/n_{e,0} = Z_i/\mu$. We see that the dust component manifests its presence through the parameter μ, defined in the previous section; recall that $\mu < 1$ ($\mu > 1$) refers to negative (positive) dust charge. As expected (and indeed verified), previous results for the IAW (in the absence of dust) [21], [25], [26] are recovered for $\mu = 1$.

B. Amplitude Modulation—Numerical Analysis

The analytical method adopted for the study of amplitude modulation and harmonic generation in the DIAW case follows the lines described in the previous section. The detailed calculation [34], [35] need not be reproduced here, since the analytical result is, in fact, exactly equivalent to the expressions obtained from the formulas presented in Section II upon setting $s = 1, \alpha \to -\tilde{\alpha}$. In particular, one thus obtains the NLS (5), which now describes the evolution of the DIAW amplitude, along with new definitions of the dispersion and nonlinearity coefficients, P and Q, obtained from (6) and (7)–(9), respectively.

As can be checked from the approximate expressions obtained for P and Q in the limit $k \ll k_D$, the wave is always stable for very long wavelengths ($\lambda \gg \lambda_D$).[2] For parallel

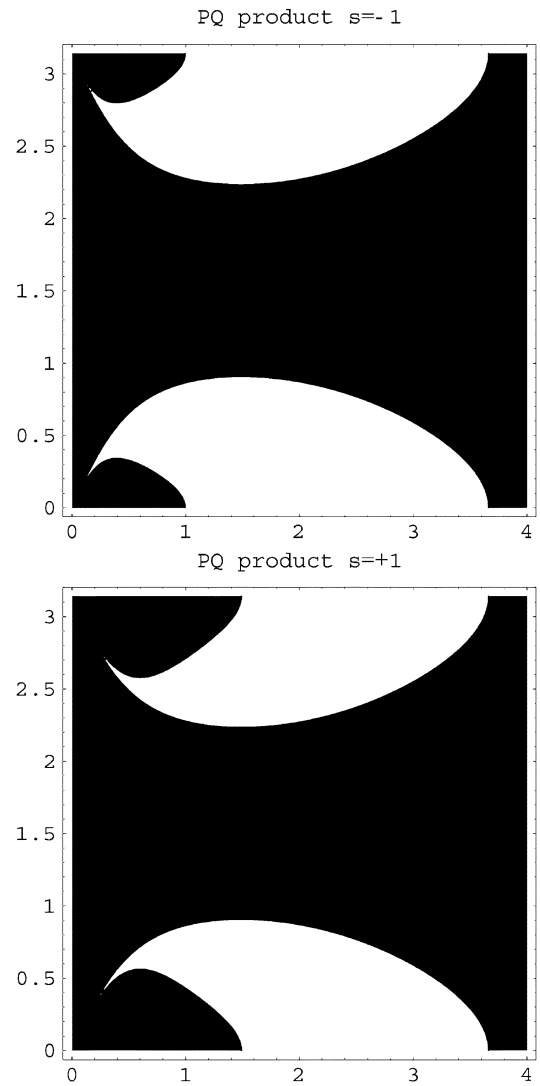

Fig. 2. Same as Fig. 1, taking $\sigma = 1$ (hot dust DAW model).

modulation ($\theta = 0$), modulational instability sets in for carrier wavenumbers k beyond a critical value, say k_{cr}; this was qualitatively expected, given the analogous studies of IAW stability (without dust) [18]–[22] (one is relieved, after such a tedious calculation, to recover exactly the previous result $k_{cr} = 1.47\, k_D$, for $\mu = 1$; see Fig. 3(a). Considering a finite ion temperature T_i (i.e., $\sigma \neq 0$), a second threshold appears, say k'_{cr}, beyond which the wave becomes stable again (corresponding plots, omitted here for brevity, are qualitatively analogous to Fig. 2). Very short wavelengths are therefore also stable in the "hot" ion model (not true for $T_i = 0$, where $k'_{cr} \to \infty$) and the instability region shrinks as k'_{cr} decreases with increasing ion temperature T_i, in agreement with IAW results [23]. The presence of negative dust ($\mu < 1$) results in a narrower stability region at low k (e.g., lower k_{cr}); negative dust therefore destabilizes long wavelength DIAW. A minimum as low as $k_{cr,\min} \approx 0.0285 k_D$ is reached around $\mu \approx 0.33$, e.g., for a negative dust charge density as high as $\delta \approx 0.67$ times the ion density. On the other hand, positive dust rather favors stability, as it slightly increases the instability threshold k_{cr} (see [34] and [35] for details).

[2]Substitute into the approximate expressions for P, Q in the low k limit (Section II) with the $\{\alpha, \beta, s\}$ parameter assignments in Section III; cf. [21, eq. (41)]: note that the factor $1/3$ therein is thus exactly recovered herein for $\mu = 1, Z_i = 1$, e.g., setting $\alpha = -\tilde{\alpha} = -1/2, \alpha' = 1/6, \beta = 1, \sigma = 0$ (and $\theta = 0$).

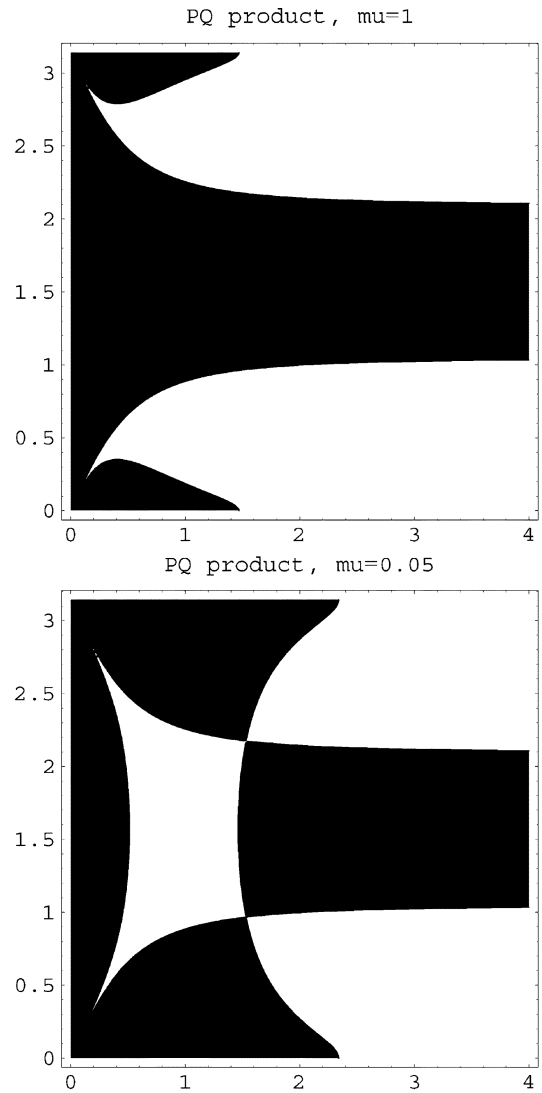

Fig. 3. $PQ = 0$ contour is depicted against k and θ (cf. Fig. 1) in two limit cases: (a) $\mu = 1$, e.g., in the dust-free (e–i plasma) limit and (b) $\mu = 0.05$, e.g., for an extremely high negative dust concentration. Notice the generation of unstable regions for high θ. Plot refers to the DIAW, setting $\sigma = 0, Z_i = 1$. We have taken $\sigma = 0$ (cold ions).

As expected from earlier predictions on IAW propagation in e–i plasma [26], obliqueness in modulation modifies the stability profile quite dramatically. The instability threshold k_{cr} is seen to decrease as θ varies from zero to some critical value θ_{cr} and then goes to infinity for higher angle values. Wide-angle modulation results in absolute stability: cf. Fig. 3(a) [however, this is not true for ultra high negative dust presence, e.g., below $\mu \approx 0.1$; see Fig. 3(b)].

Briefly concluding, oblique modulation may stabilize otherwise unstable waves and vice versa. This may also result in a shift in the type of solitary excitations sustained in the plasma, e.g., bright solitons may be destabilized when subject to oblique modulation, giving their place to their dark counterparts as preferred modes or vice versa. See [34] and [35] for a detailed study.

IV. Dust-Lattice Modes

It is now established, both theoretically and experimentally, that the increased intergrain electrostatic interaction due to

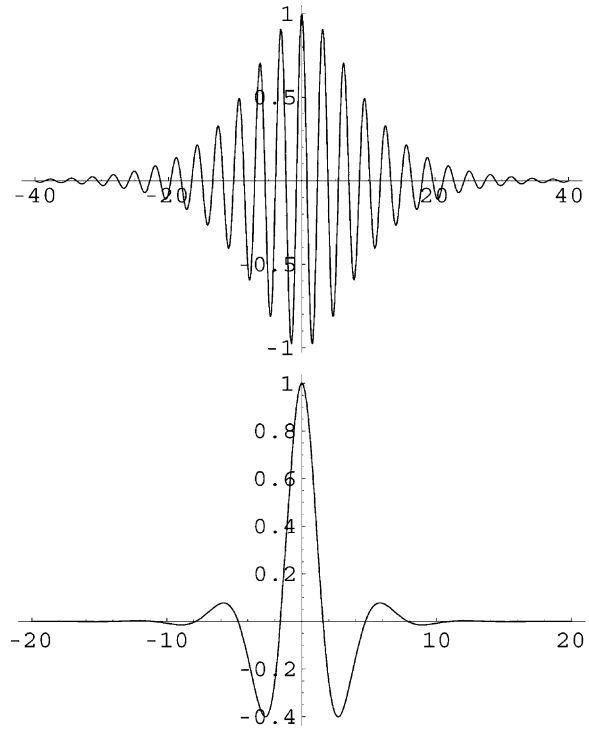

Fig. 4. Bright type (pulse) soliton solution of the NLS equation, for two different parameter sets $(PQ < 0)$. Second type, where the envelope width is not very different from the carrier wavelength, is the continuum analogue of the (discrete) breathing modes studied in molecular chains [47], [48].

the high charge acquired by dust may result in the formation of quasi-crystalline, strongly coupled dust particle periodic arrangements [1]. These dust-lattices (DL), which are typically observed at some levitated horizontal position in the sheath region above the negative electrode in discharge experiments, have been shown [9]–[13] (and experimentally observed in [14] and [15]) to support linear oscillations in both longitudinal [9]–[12] and transverse directions [13]. Because of the intrinsic chain nonlinearity, either related to the electrostatic (Debye-like, typically) interactions, in the horizontal direction, or to the sheath potential form, in the vertical one, one would *a priori* expect these excitations to be subject to a variety of nonlinear mechanisms which may underlie phenomena observed in DP crystals, e.g., instabilities, phase transitions ("melting"), etc. Yet, quite well-known in solid-state physics, such mechanisms have hardly been explored in DP modeling. Among these, modulational instability (MI) is long known to be responsible for energy localization in nonlinear lattices [44] and has been anticipated [45], [46] as an intermediate step between the linear (phonon) and strongly nonlinear (breather soliton) regime, observed in solid chains [47], [48]. Inspired by experimental evidence for nonlinear dust grain vertical oscillations in the sheath region [49], we have chosen to investigate the occurrence of the MI mechanism with respect to transverse dust-lattice waves (TDLW).

Transverse $(\sim \hat{z})$ dust-lattice waves (TDLW) propagating in a horizontal $(\sim \hat{x})$ DP crystal (lattice constant r_0) is governed by

$$M\frac{d^2\delta z_n}{dt^2} = M\omega_0^2(2\delta z_n - \delta z_{n-1} - \delta z_{n+1}) + F_z \qquad (13)$$

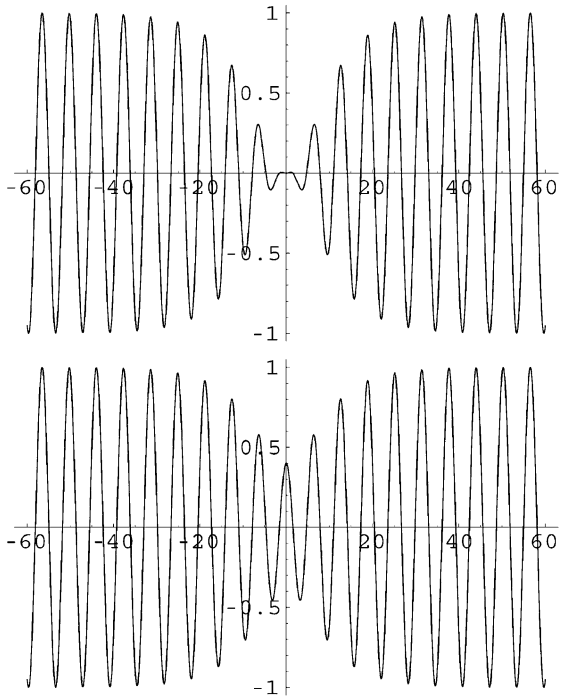

Fig. 5. Soliton solutions of the NLS equation for $PQ < 0$ (holes). Excitations are of the (a) dark type and (b) gray type. Notice that the amplitude never reaches zero in (b).

where $\delta z_n = z_n - z_0$ is the displacement of the nth grain (mass M, charge q) around the equilibrium position z_0. We have retained, for simplicity, only interactions between first neighbors. The transverse oscillation "eigenfrequency" is $\omega_0^2 = -(q/Mr_0)(\partial\Phi(x)/\partial x)|_{x=r_0}$, where $\Phi(x)$ is the intergrain interaction potential ($x = |x_i - x_j|$); e.g., for a Debye–Hückel potential form $\Phi(x) = (q/x)e^{-x/\lambda_D}$, one has $\omega_0^2 = (q^2/Mr_0^3)(1 + (r_0/\lambda_D))e^{-r_0/\lambda_D}$ (note that more sophisticated potential forms may be employed, taking into account ion flow in the sheath region [50]). The vertical force of the right-hand side $F_z = F_e - Mg$ is the result of gravity and the upward force $F_e = qE(z)$ due to the vertical electric field $E(z) = -\partial V(z)/\partial z$. The potential $V(z)$, in principle accounting for sheath and wake fields, the latter generated by downward ion flow [51], can be developed around the equilibrium position z_0 as $V(z) \approx V(z_0) + V_{(1)}\delta z + (1/2)V_{(2)}(\delta z)^2 + (1/6)V_{(3)}(\delta z)^3 + \mathcal{O}[(\delta z)^4]$; obviously, $V_{(j)} \equiv (\partial^j V(z)/\partial z^j)|_{z=z_0}$. The resulting electric force $F_e(z) = -q\sum_j (1/j!)V_{(j+1)}(\delta z)^j \equiv \sum_j \gamma_{(j)}(\delta z)^j$ now involves: a zeroth-order term, balancing gravity at z_0, a linear (restoring) term $\gamma_{(1)}\delta z \equiv -qV_{(2)}\delta z \equiv -M\omega_g^2\delta z$, and a series of higher-order (nonlinear) terms. The gap frequency ω_g may either be evaluated from first principles [13] or determined experimentally [49]. Typically, $\omega_g/2\pi \approx 20$ Hz in laboratory experiments.

By linearizing and considering $\delta z_n = A_n \exp[i(knr_0 - \omega t)] + \text{c.c.}$, we obtain the TDLW dispersion relation

$$\omega^2 = \omega_g^2 - 4\omega_0^2 \sin^2 \frac{kr_0}{2} \tag{14}$$

where ω and $k = 2\pi/\lambda$ denote, respectively, the wave frequency and the wavenumber. We will not go into further details concerning the linear regime, since it is sufficiently covered in the references. Let us now see what happens if the *nonlinear* terms are retained.

For simplicity, we shall limit ourselves to the continuum limit, considering a wavelength λ significantly larger than the intergrain distance r_0 (e.g., $kr_0 \ll 1$). The acceleration F_z/M in (13), which now describes a continuous function $\delta z = u(x,t)$, is $F_z/M = -\omega_g^2 u - \alpha u^2 - \beta u^3$. The nonlinearity coefficients α, β are related to the anharmonicity of the electric potential: $\alpha = -\gamma_{(2)}/M \equiv qV_{(3)}/(2M), \beta = -\gamma_{(3)}/M \equiv qV_{(4)}/(6M)$.

Considering small displacements $u = \epsilon u_1 + \epsilon^2 {u_2}^2 + \cdots$ and employing the reductive perturbation technique described above, we obtain a solution of the form

$$u(x,t) = \epsilon\left[Ae^{i(kx-\omega t)} + \text{c.c.}\right] + \epsilon^2\alpha\left[-\frac{2|A|^2}{\omega_g^2} + \frac{A^2}{3\omega_g^2}e^{2i(kx-\omega t)} + \text{c.c.}\right] + \mathcal{O}(\epsilon^3) \tag{15}$$

where ω obeys the relation $\omega^2 = \omega_g^2 - c_0^2k^2$ (where $c_0 = \omega_0 r_0$). Precisely, it is (14) near $k \approx 0$. The slowly varying amplitude $A = A(\zeta)$ moves at the (negative) group velocity $v_g = d\omega/dk = -c_0^2k/\omega$ in the direction *opposite* to the phase velocity. This *backward* wave has been observed experimentally [52]. The amplitude A obeys the NLSE (5) where the "slow" variables are $\{\zeta, \tau\}$ and $\{\epsilon(x - v_g t), \epsilon^2 t\}$. The NLSE coefficients are now given by

$$P = \omega''(k)/2 = -c_0^2\omega_g^2/(2\omega^3)$$

and

$$Q = \frac{1}{2\omega}\left(\frac{10\alpha^2}{3\omega_g^2} - 3\beta\right) = \frac{\omega_g^2}{4\omega}\left[\frac{5}{3}\left(\frac{V_3}{V_2}\right)^2 - \frac{V_4}{V_2}\right].$$

One is now left with the task of evaluating P and Q for a given explicit form of $V(z)$ and then investigating the TDL wave stability. The exact form of the potential $V(z)$ may be obtained from experimental data fitting. For instance, in [49], the dust grain potential energy $\mathcal{U}(z) = qV(z)$ was reconstructed from experimental data as $\mathcal{U}(z) \simeq M\omega_g^2[-0.9\delta z + (1/2)(\delta z)^2 - (1/3)0.5(\delta z)^3 + (1/4)0.07(\delta z)^4 + \cdots]$; see (9) therein. Upon simple comparison with our definitions above, we obtain $V_{(3)}/V_{(2)} = -1\text{mm}^{-1}, V_{(4)}/V_{(2)} = 0.42\text{mm}^{-2}$,, so Q is positive. Now see that $P < 0$, given the parabolic form of $\omega(k)$ close to $k = 0$ (continuum limit). Therefore, the transverse oscillation considered in [49] would propagate as a *stable* wave, for large wavelength values λ. However, for shorter wavelengths, e.g., in the general (discrete) case, the coefficient $P = \omega''(k)/2$ changes sign at some critical value of k (the *zero-dispersion point*) in the first Brillouin zone; see (14). So, P becomes positive and then so does the product PQ; the TDL wave may thus be potentially unstable for higher wavenumbers.

Since the TDL wave amplitude evolution is described by the NLSE (5), the existence of either pulse-shaped localized solutions (*bright* envelope solitons, for $PQ > 0$) or *dark/gray* ($PQ < 0$) type [42], [43] may be anticipated; their analytical form is given above. The former (*continuum breathers*) may

occur and propagate in a DP crystal if a sufficiently short wavelength is chosen (so that $PQ > 0$). The latter (*holes*), yet apparently privileged in the continuum limit, are rather physically irrelevant in this (infinite chain) model, since they correspond to infinite energy stored in the lattice. These predictions may, in principle, be checked experimentally.

V. Conclusion

Summarizing our results, we have seen the following.

1) Electrostatic waves propagating in a complex plasma are subject to modulational instability.
2) Allowing for an oblique wave amplitude modulation strongly affects the conditions for instability to occur; regions which are stable to a parallel modulation may become unstable when subject to an oblique modulation and vice versa.
3) Large-angle θ modulation seems to have a stabilizing effect, while small-to-medium θ values rather promote instability.
4) Envelope localized excitations may form and propagate in a dusty plasma; modulationally stable (unstable) values of (k, θ) sustain solitary waves of the bright (dark) type i.e., potential humps (dips), the type and characteristics of which depend on k, θ via the NLS equation coefficients.
5) In comparison to the dust-free (e–i) plasma case, the presence of positive (negative) dust rather enhances (impedes) stability and slightly favors dark (bright) type excitations, e.g., holes (pulses).

These results may be confirmed by appropriate experiments and/or numerical investigations (beyond the scope of this paper).

Our aim has been to put forward a generic model for the study of DP mode-related amplitude modulation and harmonic generation. We have studied the modulational instabilities and associated envelope soliton formation. Besides modulational instabilities, the dust ion-acoustic and transverse dust lattice waves could be subject to a decay instability in which a pump wave decays into a daughter wave and an ultralow-frequency (ULF) dust acoustic wave [53] or, presumably, a longitudinal ULF dust lattice wave, respectively. Thus, in the decay interaction, the lower sideband and the ULF waves are resonant, contrary to nonresonant zero-frequency modulating perturbations producing modulational instabilities. Clearly, the underlying physics of the decay and modulational instabilities are different and they will appear in different parameter regimes. Focusing on the qualitative aspects of this paper, we have ignored effects such as dust charge fluctuations and the ion drag force. It should, however, be noted that dust charge fluctuations usually produce non-Landau damping [1] of DA and DIA waves, leading to the formation of nonenvelope DA and DIA shocks [54], [55]. On the other hand, the ion drag force acting on the dust grains can produce linear instabilities [1] of DA and DIA waves depending on the dust grain radius. However, in the present paper, we have dealt with stable DA, DIA, and transverse dust lattice waves, which undergo nonlinear instabilities due to self-interactions of carrier waves.

References

[1] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics*. Bristol, MA: Inst. Physics, 2002.
[2] F. Verheest, *Waves in Dusty Space Plasmas*, Dordrecht, Germany: Kluwer, 2001.
[3] N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics*. New York: McGraw-Hill, 1973.
[4] Th. Stix, *Waves in Plasmas*. New York: AIP, 1992.
[5] P. K. Shukla and V. P. Silin, "Dust ion-acoustic wave," *Phys. Scr.*, vol. 45, p. 508, 1992.
[6] N. N. Rao, P. K. Shukla, and M. Y. Yu, "Dust-acoustic waves in dusty plasmas," *Planet. Space Sci.*, vol. 38, p. 543, 1990.
[7] A. Barkan, R. Merlino, and N. D'Angelo, "Laboratory observation of the dust acoustic wave mode," *Phys. Plasmas*, vol. 2, no. 10, p. 3563, 1995.
[8] J. Pieper and J. Goree, "Dispersion of plasma dust acoustic waves in the strong coupling regime," *Phys. Rev. Lett.*, vol. 77, p. 3137, 1996.
[9] F. Melandsø, "Lattice waves in dust plasma crystals," *Phys. Plasmas*, vol. 3, p. 3890, 1996.
[10] B. Farokhi, P. K. Shukla, N. L. Tsintsadze, and D. D. Tskhakaya, "Linear and nonlinear dust lattice waves in plasma crystals," *Phys. Lett. A*, vol. 264, p. 318, 1999.
[11] ——, "Dust lattice waves in plasma crystals," *Phys. Plasmas*, vol. 7, p. 814, 2000.
[12] D. D. Tskhakaya and P. K. Shukla, "Theory of dust crystal and its oscillations in plasmas," *Phys. Lett. A*, vol. 286, p. 277, 2001.
[13] S. V. Vladimirov, P. V. Shevchenko, and N. F. Cramer, "Vibrational modes in the dust-plasma crystal," *Phys. Rev. E*, vol. 56, p. R74, 1997.
[14] A. Melzer, T. Trottenberg, and A. Piel, "Experimental determination of the charge on dust particles forming Coulomb lattices," *Phys. Lett. A*, vol. 191, p. 301, 1994.
[15] Y. Hayashi and K. Tachibana, "Observation of Coulomb-crystal formation from carbon particles grown in a methane plasma," *Jpn. J. Appl. Phys.*, vol. 33, p. L804, 1994.
[16] M. Remoissenet, *Waves Called Solitons*. Berlin, Germany: Springer-Verlag, 1994.
[17] A. Hasegawa, *Optical Solitons in Fibers*. Berlin, Germany: Springer-Verlag, 1989.
[18] T. Taniuti and N. Yajima, "Perturbation method for a nonlinear wave modulation," *J. Math. Phys.*, vol. 10, p. 1369, 1969.
[19] N. Asano, T. Taniuti, and N. Yajima, "Perturbation method for a nonlinear wave modulation II," *J. Math. Phys.*, vol. 10, p. 2020, 1969.
[20] T. Kakutani and N. Sugimoto, "Krylov-Bogolibov-Mitropolsky method for nonlinear wave modulation," *Phys. Fluids*, vol. 17, p. 1617, 1974.
[21] K. Shimizu and H. Ichikawa, "Automodulation of ion oscillation modes in plasma," *J. Phys. Soc. Jpn.*, vol. 33, p. 789, 1972.
[22] M. Kako, "Part III. Applications to the collisionless plasma in fluid model. B. Nonlinear modulation of plasma waves," *Prog. Theor. Phys. Suppl.*, vol. 55, p. 120, 1974.
[23] I. R. Durrani, G. Murtaza, H. U. Rahman, and I. A. Azhar, "Effect of ionic temperature on the modulational instability of ion acoustic waves in a collisionless plasma," *Phys. Fluids*, vol. 22, p. 791, 1979.
[24] V. S. Chan and S. R. Seshadri, "Modulational stability of the ion plasma mode," *Phys. Fluids*, vol. 18, p. 1294, 1975.
[25] J.-K. Xue, W.-S. Duan, and L. He, "Modulational instability of ion-acoustic waves in a warm plasma," *Chin. Phys.*, vol. 11, p. 1184, 2002.
[26] M. Kako and A. Hasegawa, "Stability of oblique modulation on an ion-acoustic wave," *Phys. Fluids*, vol. 19, p. 1967, 1976.
[27] R. Chhabra and S. Sharma, "Modulational instability of obliquely modulated ion-acoustic waves in a two-ion plasma," *Phys. Fluids*, vol. 29, p. 128, 1986.
[28] M. K. Mishra, R. S. Chhabra, and S. R. Sharma, "Stability of oblique modulation of ion-acoustic waves in a multicomponent plasma," *Phys. Plasmas*, vol. 1, p. 70, 1994.
[29] Y. H. Ichikawa, T. Imamura, and T. Taniuti, "Nonlinear wave modulation in collisionless plasmas," *J. Phys. Soc. Jpn.*, vol. 33, p. 189, 1972.
[30] H. Sanuki, K. Shimizu, and J. Todoroki, "Effects of Landau damping on nonlinear wave modulation in plasma," *J. Phys. Soc. Japan*, vol. 33, p. 198, 1972.
[31] Y. H. Ichikawa and T. Taniuti, "Nonlinear wave modulation with account of the nonlinear Landau damping," *J. Phys. Soc. Jpn.*, vol. 34, p. 513, 1973.
[32] W. Duan, K. Lü, and J. Zhao, "Hot dust acoustic solitary waves in dust plasma with variable dust charge," *Chin. Phys. Lett.*, vol. 18, p. 1088, 2001.

[33] I. Kourakis and P. K. Shukla, "Oblique amplitude modulation of dust-acoustic plasma waves," *Phys. Scripta*, vol. 69, no. 4, p. 316, 2004.
[34] I. Kourakis and P. K. Shukla, "Modulational instability and localized excitations of dust-ion acoustic waves," *Phys. Plasmas*, vol. 10, no. 9, p. 3459, 2003.
[35] ——, "Finite ion temperature effects on the stability and envelope excitations of dust-ion acoustic waves," *Eur. Phys. J. D*, vol. 28, p. 109, 2004.
[36] M. R. Amin, G. E. Morfill, and P. K. Shukla, "Modulational instability of dust-acoustic and dust-ion-acoustic waves," *Phys. Rev. E*, vol. 58, p. 6517, 1998.
[37] M. R. Amin, G. E. Morfill, and P. K. Shukla, "Amplitude modulation of dust-lattice waves in a plasma crystal," *Phys. Plasmas*, vol. 5, no. 7, p. 2578, 1998.
[38] ——, "Modulational instability of dust-lattice waves in a plasma crystal," *Phys. Scripta*, vol. 58, p. 628, 1998.
[39] I. Kourakis, "Weakly nonlinear effects associated with transverse oscillations in dusty plasma crystals," *Phys. Scripta*, vol. T107, p. 243, 2004.
[40] A. Hasegawa, *Plasma Instabilities and Nonlinear Effects*. Berlin, Germany: Springer-Verlag, 1975.
[41] R. Fedele, "Envelope solitons versus solitons," *Phys. Scripta*, vol. 65, p. 502, 2002.
[42] R. Fedele, H. Schamel, and P. K. Shukla, "Solitons in the Madelung's fluid," *Phys. Scripta*, vol. T98, p. 18, 2002.
[43] R. Fedele and H. Schamel, "Solitary waves in the Madelung's fluid: A connection between the nonlinear Schrödinger equation and the Korteweg-de Vries equation," *Eur. Phys. J. B*, vol. 27, p. 313, 2002.
[44] Y. S. Kivshar and M. Peyrard, "Modulational instabilities in discrete lattices," *Phys. Rev. A*, vol. 46, p. 3198, 1992.
[45] I. Daumont, T. Dauxois, and M. Peyrard, "Modulational instability: First step toward energy localization in nonlinear lattices," *Nonlinearity*, vol. 10, p. 617, 1997.
[46] M. Peyrard, "The pathway to energy localization in nonlinear lattices," *Phys. D*, vol. 119, p. 184, 1998.
[47] G. Tsironis and E. N. Economou, Eds., *Fluctuations, Disorder and Nonlinearity, Phys. D*. Amsterdam, The Netherlands: North-Holland, 1998, vol. 113.
[48] T. Bountis, H. W. Capel, and M. Kollmann, "Multibreathers and homoclinic orbits in 1-dimensional nonlinear lattices," *Phys. Lett. A*, vol. 268, p. 50, 2000.
[49] A. V. Ivlev *et al.*, "Nonlinear vertical oscillations of a particle in a sheath of a rf discharge," *Phys. Rev. Lett.*, vol. 85, p. 4060, 2000.
[50] I. Kourakis and P. K. Shukla, "Study of the intergrain interaction potential and associated instability of dust-lattice plasma oscillations in the presence of ion flow," *Phys. Lett. A*, vol. 317, no. 1–2, p. 156, 2003.
[51] P. K. Shukla and N. N. Rao, "Coulomb crystallization in colloidal plasmas with streaming ions and dust grains," *Phys. Plasmas*, vol. 3, p. 1770, 1996.
[52] T. Misawa *et al.*, "Experimental observation of vertically polarized transverse dust-lattice wave propagating in a one-dimensional strongly coupled dust chain," *Phys. Rev. Lett.*, vol. 86, p. 1219, 2001.
[53] P. K. Shukla, Y. Nakamura, N. Nambu, and S. V. Vladimirov, "Nonlinear coupling between fast and slow dust-acoustic waves," *J. Phys. Soc. Jpn.*, vol. 64, no. 7, p. 2278, 1995.
[54] F. Melandsø and P. K. Shukla, "Theory of dust-acoustic shocks," *Planet. Space Sci.*, vol. 43, p. 635, 1995.
[55] A. A. Mamun and P. K. Shukla, "The role of dust charge fluctuations on nonlinear dust ion-acoustic waves," *IEEE Trans. Plasma Sci.*, vol. 30, pp. 720–724, Apr. 2002.
[56] I. Kourakis, *Proc. 29th EPS Meeting Controlled Fusion and Plasma Physics Eur. Physical Soc.*, 2002.

Ioannis Kourakis was born in Chania, Crete, Greece. He received the B.Sc. degree in physics from the University of Crete, Heraklion, Greece, the D.E.A. (French M.Sc. equivalent) degree in molecular spectroscopy and material science from the University of Burgundy, Dijon, France, the D.E.A. (Belgian M.Phil. equivalent) degree in science, and the Ph.D. degree in theoretical physics, both from the Free University of Brussels (U.L.B.), Brussels, Belgium.

He has carried out research and teaching work at the University of Burgundy and the Free University of Brussels and is currently a Research Fellow at the Ruhr-Universitaet Bochum, Bochum, Germany. His research interests include the physics of plasmas (dusty-complex plasmas, fusion plasmas), solid state physics (theoretical), statistical mechanics (kinetic theory), and nonlinear dynamics (nonlinear coherent excitations, wave propagation in nonlinear dispersive media).

Dr. Kourakis is a member of the American Physical Society, Belgian Physical Society, and French Physical Society.

Padma K. Shukla received the Ph.D. degree in physics from Banaras Hindu University, Varanasi, India, and the Ph.D. degree in plasma physics from Umeå University, Umeå, Sweden, in 1972 and 1975, respectively.

He moved from India to Umeå in January 1972 and later (in January 1973) joined the Faculty of Physics and Astronomy, Ruhr University Bochum, Germany. He also became a Professor with the Department of Plasma Physics, Umeå University, in December 1997. His main research interests include theoretical plasma physics, nonlinear structures in plasmas and fluids, collective processes in space physics, and astrophysics. His present interests involve the physics of dusty plasmas, astro-particle physics (collective neutrino plasma interaction), and nonlinear phenomena in planetary magnetospheres and astrophysics. He is an Associate Editor of the *Journal of Plasma Physics*. Currently, he is also serving as an Editorial Board Member of *Plasma Physics and Controlled Fusion* and the *Journal of Research in Physics*.

Dr. Shukla is an Associate Editor with the IEEE TRANSACTIONS ON PLASMA SCIENCE. He has also been honored as a Fellow of the American Physical Society and the Institute of Physics (U.K.).